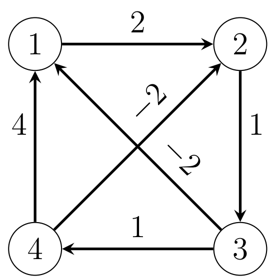
Math 482 **HW9** **4/13/2014** Name: ______________________

Due Friday, April 18, 2014

Students in section B13 (three credit hours) need to solve any four of the following five problems. Students in section B14 (four credit hours) must solve all five problems.

1. Find the length of a shortest paths from every vertex to all other vertices in the graph above (right) using the Floyd–Warshall algorithm. Your answer should include the matrix D^k for every k from 0 to 4. (This problem is counted as two problems. For one-credit, you only need to compute the matrices D^0, D^1 and D^2.)



2. Let $G = (V, E)$ be a directed graph such that $V = \{1, \dots, n\}$, let $w : E \to \mathbb{R}$ be a weight function and suppose that G has no negative weight cycles. Let E^0 be the $n \times n$ matrix in which each entry $e^0_{i,j} = 0$. While the Floyd-Warshall algorithm is applied to G, we let

$$e^k_{i,j} = \begin{cases} k & \text{if } d^{k-1}_{i,k} + d^{k-1}_{k,j} < d^{k-1}_{i,j} \\ e^{k-1}_{i,j} & \text{otherwise} \end{cases}$$

and E^k be the $n \times n$ matrix with entries $\{e^k_{i,j}\}$. Describe how the matrix $E = E^n$ can be used to reconstruct a shortest path between any two vertices in G.

3. Consider the following integer program P:

$$\begin{array}{rcccl} & z = x_1 & \to & \text{min} & \\ \text{subject to} & 3x_1 & -100x_2 & \geq & 1 \\ & 3x_1 & -101x_2 & \leq & 1 \\ & x_1, & x_2 & \geq & 0 \\ & x_1, & x_2 & & \text{integer} \end{array}$$

Solve the linear programming relaxation of P, obtaining an optimal solution x^* with cost z^* (You do not need to use the simplex algorithm to solve this problem). Obtain an integer vector x from x^* by rounding each component to the nearest integer. Is x an optimal solution to the integer program P? If it is not, find an optimal solution to the integer program P.

4. Let $G = (V, E)$ be an undirected graph. A set $U \subseteq V$ is an independent set in G, if there does not exist an edge $\{v_i, v_j\} \in E$ such that both $v_i \in U$ and $v_j \in U$. Formulate the problem of finding a maximum size independent set in G as an integer linear program.