Math 482 **HW6** **3/08/2014** Name: ____________________

Due Friday, March 14, 2014

Students in section B13 (three credit hours) need to solve any four of the following five problems. Students in section B14 (four credit hours) must solve all five problems.

1. Use the complementary slackness condition to check whether the vector $[3,\ -1,\ 0,\ 2]^T$ is an optimal solution to the problem

$$\text{Maximize } z = 6x_1 + x_2 - x_3 - x_4$$

subject to

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 & \leq & 5, \\ 3x_1 + x_2 - x_3 & \leq & 8, \\ x_2 + x_3 + x_4 & = & 1, \\ x_3, x_4 & \geq & 0. \end{cases}$$

2. Let A be an $m \times n$ matrix of rank m and let P be the following LP in standard form.

$$\text{Minimize } z = c^T x$$
$$\text{subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

Prove that if the P has an optimal solution and P has no degenerate optimal solutions, then there is a unique optimal solution to the dual of P. Recall that a degenerate solution is a basic feasible solution with more than $n - m$ zeros. (Hint: Use the complementary slackness condition and the fact that if an LP in standard form has an optimal solution, then it has an optimal basic feasible solution)

3. Use the revised simplex method to find an optimal solution to the problem

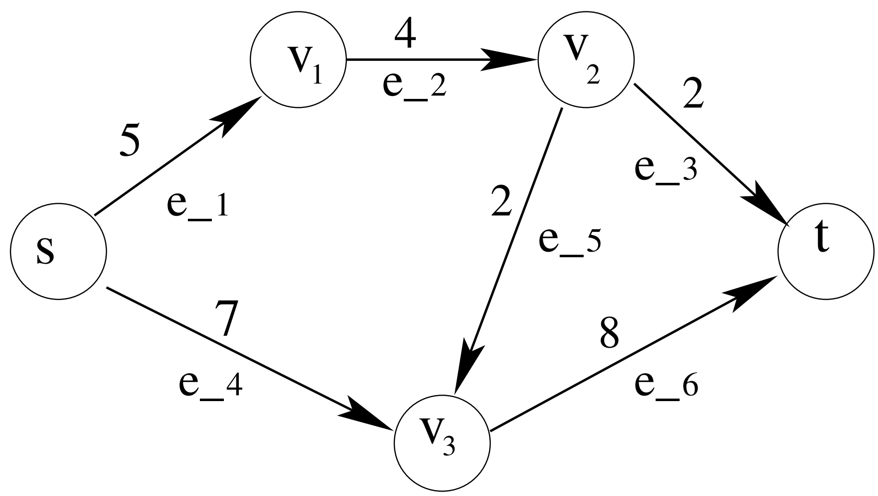
$$\text{Minimize } z = x_1 + x_2 + x_3$$

subject to

$$\begin{cases} x_1 + x_4 - 2x_6 & = & 5, \\ x_2 + 2x_4 - 3x_5 + x_6 & = & 3, \\ x_3 + 2x_4 - 3x_5 + 6x_6 & = & 5, \\ x_1, \ldots, x_6 & \geq & 0. \end{cases}$$

4. THIS PROBLEM IS COUNTED AS TWO PROBLEMS. TO EARN ONE PROBLEM CREDIT, IT IS ENOUGH TO PERFORM TWO ITERATIONS CORRECTLY.

   Use the revised simplex method to find a maximal flow in the network below. **Do not** write explicitly the whole matrix. Use auxiliary shortest path problems to find a pivot column.



4
v_1
e_2
v_2
2
5
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