Math 482 **HW5** **2/28/2014** Name: ____________________

Due Friday, March 7, 2014

Students in section B13 (three credit hours) need to solve any four of the following five problems. Students in section B14 (four credit hours) must solve all five problems.

1. Suppose that there exists x_0 and y such that x_0 is feasible for the linear program P

$$\begin{array}{rrcl} \min & c^T x & & \\ \text{subject to} & Ax & = & b \\ & x & \geq & 0 \end{array}$$

   and y satisfies

$$c^T y < 0$$
$$Ay = 0$$
$$y \geq 0.$$

   Prove that P is unbounded.

2. Let the matrix define a game and let Alice be the player whose pure strategies are represented by the rows of the matrix and Bob be the player whose pure strategies are represented by the columns of the matrix. What is the optimal pure strategy for Alice and what is expected payout given that choice? In other words, if Alice must play the same pure strategy in every turn of the game, what pure strategy should she play and what is the expected payout assuming Bob plays optimally? Determine the same information for Bob.

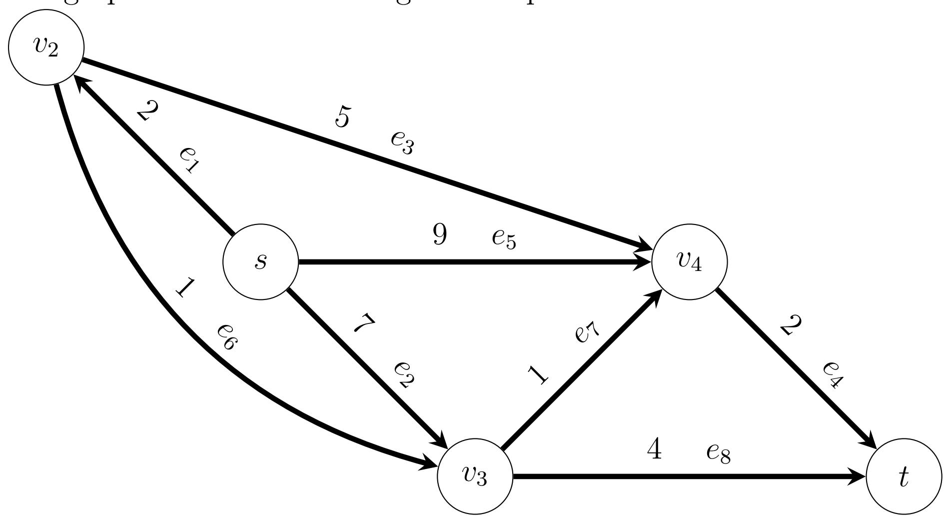
$$\begin{pmatrix} 4 & 8 & 1 & 2 & 6 & 7 \\ 8 & 2 & 3 & 5 & 9 & 2 \\ 1 & 8 & 1 & 7 & 3 & 3 \\ 6 & 5 & 6 & 8 & 6 & 3 \\ 2 & 6 & 2 & 4 & 7 & 1 \\ 3 & 9 & 1 & 5 & 2 & 6 \end{pmatrix}.$$

3. Solve the game with the payoff matrix

$$\begin{pmatrix} 3 & 5 & 3 & -2 & 0 \\ 3 & 7 & 3 & -1 & 1 \\ 2 & -4 & 1 & 3 & 4 \\ 1 & -5 & 1 & 3 & 0 \end{pmatrix}$$

   using the simplex method. Here both players are allowed to use mixed strategies. To make the problem simpler, iteratively delete rows and columns that correspond to pure strategies that clearly will not be played in any optimal solution before constructing and solving the associated LP.

4. Interpreting the numbers on edges as edge lengths, solve the shortest path problem for the graph drawn below using the simplex method with the initial basis $\{e_1, e_2, e_3, e_4\}$.



5. State the dual to the shortest path problem above and give a solution. (You can use the solution to the previous problem and complementary slackness or any other ideas.)