Math 482 **HW4** **2/21/2014** Name: ____________________

Due Friday, February 28, 2014

Students in section B13 (three credit hours) need to solve any four of the following five problems. Students in section B14 (four credit hours) must solve all five problems.

1. State the dual to the following problem:

$$z = 3x_1 - x_2 - 2x_3 \quad \longrightarrow \quad \min$$

with respect to

$$\begin{cases} 6x_1 & -2x_2 & +3x_3 & & +2x_5 & \geq & -7, \\ -2x_1 & +3x_2 & & -2x_4 & & \leq & 6, \\ 2x_1 & +x_2 & -4x_3 & +x_4 & & = & 6, \\ & & & x_1, & x_2 & x_4 \geq & 0 \end{cases}$$

2. The Transportation Problem (known also as the Hitchcock problem) is as follows. There are m sources of some commodity, each with a supply of a_i units, $i = 1, \dots, m$ and n terminals, each of which has a demand of b_j units, $j = 1, \dots, n$. The cost of sending a unit from source i to terminal j is c_{ij} and $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$. We want to find a cheapest way to satisfy all demands. State this problem as an LP, and state the dual to this problem.

3. Use the dual simplex method to find an optimal solution to the problem

$$\text{Minimize } z = 7x_1 + x_2 + 3x_3 + x_4$$

subject to

$$\begin{cases} 2x_1 - 3x_2 - x_3 + x_4 & \geq & 8, \\ 6x_1 + x_2 + 2x_3 - 2x_4 & \geq & 12, \\ -x_1 + x_2 + x_3 + x_4 & \geq & 10, \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{cases}$$

4. The additional constrains

$$\begin{aligned} x_1 + 5x_2 + x_3 + 7x_4 &\leq 50, \\ 3x_1 + 2x_2 - 2x_3 - x_4 &\leq 20 \end{aligned}$$

are added to those of Problem 3. Solve the new problem starting from the optimal tableau for Problem 3.

5. Prove the theorem due to P. Gordan (1873) that the system $\mathbf{Ax} < \mathbf{0}$ is unsolvable if and only if the system $\mathbf{y^T A} = \mathbf{0}, \quad \mathbf{y} \geq \mathbf{0}, \quad \mathbf{y} \neq \mathbf{0}$ is solvable. (Hint: In order to apply duality theorems, replace the system $\mathbf{Ax} < \mathbf{0}$ of strict inequalities by the system $\mathbf{Ax} \leq \mathbf{-1}$ of nonstrict inequalities. Prove that the new system is solvable if and only if $\mathbf{Ax} < \mathbf{0}$ is solvable).