Math 412 HW8 4/06/2014 Name: ____

Due Friday, April 11, 2014

Students in section X13 (three credit hours) need to solve any four of the following five problems. Students in section X14 (four credit hours) must solve all five problems.

- 1. Let G be a n-vertex 3-regular graph with at most 5 cut-edges. Prove that G has a matching with at least $\frac{n}{2} 1$ edges.
- 2. # 4.1.11 in the book.
- 3. Let *l* be odd and let *G* be a 3-regular graph graph such that $n(G) \leq 2l$, $\kappa'(G) \geq 2$ and every cycle in *G* has length at least *l*. Use Propositions 4.1.9, 4.1.11 and 4.1.12 to prove that $\kappa(G) = 3$. Note that this implies that the Peterson graph is 3-connected.
- 4. Prove that every 2-connected graph G has a cycle of length at least min $\{n(G), 2\kappa G\}$.
- 5. For any graph G a Hamiltonian cycle in G is a subgraph of G that is a cycle and has length n(G). Suppose G is a 2-connected graph with $\alpha(G) \leq \kappa(G)$. Prove that G has a Hamiltonian cycle.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS:

Section 4.1: # 1, 2, 3, 5, 7. Section 4.2: # 1, 3, 4. OTHER INTERESTING PROBLEMS: Section 4.1: # 10, 18, 24.

Section 4.2: # 8. Do not write these up!