The following are some sample questions for exam 1. Some of these questions may appear on the exam 1. There will be more questions on the exam than I have listed here.

Question 1:
Write the definitions of the following terms :
- an indefinite matrix
- $B(x, r)$, the open ball centered at $x \in \mathbb{R}^n$ with radius $r \in \mathbb{R}$ such that $r > 0$
- the function $f : \mathbb{R}^n \to \mathbb{R}$ is *coercive*

Question 2:

(a) *Complete the statement of Taylor's formula for real valued functions on* $\mathbb{R}^n$.

Suppose that x^*, x are points in $\mathbb{R}^n$ and that $f(x)$ is a function from $\mathbb{R}^n$ to $\mathbb{R}$ with continuous first and second partial derivatives on some open set containing the line segment $[x^*, x] = \{w \in \mathbb{R}^n : w = x^* + t(x - x^*); 0 \le t \le 1\}$ joining x^* and x. Then there exists ....

(b) Let $f(x)$ be a real valued convex function defined on a convex subset $C \subseteq \mathbb{R}^n$ and let $M \subseteq C$ be the set of local minimizers of f on C. Which of the following is true about the set M.

A. The set M is not empty.

B. Every point in M is a critical point of f.

C. Every point in M is a global minimizer of f on D.

D. For every $x \in M$, $Hf(x)$ is defined and is positive semidefinite.

E. All of the above

Question 3:
Determine if the following matrix is positive/negative (semi)definite or indefinite (you must justify your answer):

$$\begin{pmatrix} -4 & 0 & 1 \\ 0 & -3 & 2 \\ 1 & 2 & -5 \end{pmatrix}$$

Question 4:
Let $B \in \mathbb{R}^{n \times n}$ and $\mathbf{b} \in \mathbb{R}^n$ be fixed. Define $D = \{\mathbf{x} \in \mathbb{R}^n : B\mathbf{x} = \mathbf{b}\}$. Is D convex? *(You must prove your answer is correct).*

Question 5:
Find (local, global) minimizers and maximizers of the following function (you must justify your answers):

(a) $f(x, y) = 2x^2 - 4xy + 3y^2 - 6x - 12y$ for $(x, y) \in \mathbb{R}^2$

Question 6:
Determine whether the following function is convex, concave, strictly convex or strictly concave on $(x_1, x_2) \in \mathbb{R}^2$ (you must justify your answer):

$$f(x_1, x_2) = 3 + 3x_1 - 10x_2 + e^{\left(x_1^2 - 2x_1x_2 + 2x_2^2\right)}$$

A questions like the following will only be required for C14 students. It would be optional for C13 students

Question 7: *(C14 required, C13 optional)*
State and prove the theorem about coercive functions and global minimizers.