Math-484 Homework #10 (iterative methods)

Complete to prepare for the final exam For all of the following problems, you you are allowed and encouraged to use software for computing product of matrices, inverses of matrices.

1: Compute the first two terms $\mathbf{x}_1, \mathbf{x}_2$ of the Newton's Method sequence $\{\mathbf{x}_k\}$ for minimizing the function

$$f(x_1, x_2) = 2x_1^4 + x_2^2 - 4x_1x_2 + 5x_2$$

with initial point $\mathbf{x}_0 = (0, 0)$.

2: Show that the function $f(x) : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = |x|^{4/3}$$

has a unique global minimizer at $x^* = 0$ but that, for any nonzero initial point x_0, the Newton's Method sequence $\{x_k\}$ with initial point x_0 for minimizing $f(x)$ diverges.
Hint: Solve for $x_k > 0$ and use symmetry.

3: (a) Compute the quadratic approximation $q(\mathbf{x})$ for

$$f(x_1, x_2) = 8x_1^2 + 8x_2^2 - x_1^4 - x_2^4 - 1$$

at the point $(\frac{1}{2}, \frac{1}{2})$.
Note the quadratic approximation f at $\mathbf{x}^*$ is

$$q(\mathbf{x}) = f(\mathbf{x}^*) + \nabla f(\mathbf{x}^*) \cdot (\mathbf{x} - \mathbf{x}^*) + \frac{1}{2}(\mathbf{x} - \mathbf{x}^*) \cdot Hf(\mathbf{x})(\mathbf{x} - \mathbf{x}^*).$$

(b) Compute the minimum $\mathbf{x}^*$ of the quadratic approximation $q(\mathbf{x})$ at $(\frac{1}{2}, \frac{1}{2})$.

4: Compute the first two terms $\mathbf{x}_1, \mathbf{x}_2$ of the Steepest Descent sequence $\{\mathbf{x}_k\}$ for minimizing the function

$$f(x_1, x_2) = 2x_1^4 + x_2^2 - 4x_1x_2 + 5x_2$$

with initial point $\mathbf{x}_0 = (0, 0)$.

5: Prove that if A is a positive definite matrix, then A^{-1} exists and in positive definite.
(Recall that A is also symmetric since it is part of being positive definite.)