Math-484 Homework #9 (minimization with equality constraints)
Due 10am Apr 21.

1: Determine all maxima and minima of

$$f(x_1, x_2, x_3) = x_1 x_3 + x_2^2$$

on the sphere $x_1^2 + x_2^2 + x_3^2 = 4$.

2: Problem 2 removed

3: Sketch the feasible region of the following convex program and show that the point $(1, 0)$ is feasible but is not a regular point.

$$(P) \begin{cases} \text{Minimize} & f(x_1, x_2) \\ \text{subject to} & -1 \le x_1 \le 3 \\ & x_2 \ge 0 \\ & x_2 - (x_1 - 1)^2 \le 0. \end{cases}$$

4: The output of a manufacturing operation is a quantity Q which is a function $Q = Q(x, y)$ where $x =$ capital equipment and $y =$ hours of labor. You can assume that Q has continuous first partial derivatives. Suppose the price of labor is p and the price of investment is q in dollars and the operation is to spend exactly b dollars. For optimum production, we want to maximize Q subject to $qx + py = b$. Show that at an optimum, we have

$$\frac{\partial Q}{\partial K} = \frac{\partial Q}{\partial l},$$

where $K = qx$ and $l = py$. Thus, at an optimum, the marginal change in output per dollar's worth of additional capital equipment is equal to the marginal change in output per dollar's worth of additional labor.

5: For the following program

$$(P) \begin{cases} \text{Minimize} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} & (x_1 - 2)^3 - x_2^2 = 0, \end{cases}$$

a) Sketch the feasible region and determine the solution graphically.

b) Show that the program admits no Lagrange multipliers.

c) Explain why this does not violate theorem 7.2.1