Math-484 Homework #8 (penalty method)
Due 10am Apr 14.

1: Consider the following program:

$$(P)\begin{cases}\text{Minimize} & f(x) = x^2 - 2x\\ \text{subject to} & 0 \le x \le 1.\end{cases}$$

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for (P).
(b) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $P_k(x)$ with the Courant-Beltrami Penalty Term.
(c) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $F_k(x)$ with the Absolute Value Penalty Term.

2:

a) Use the penalty function method with the Courant-Beltrami penalty term to solve the problem (P).

$$(P)\begin{cases}\text{Minimize} & f(x_1, x_2) = x_1 + x_2\\ \text{subject to} & x_1^2 - x_2 \le 2\end{cases}$$

b) Show that the objective function $F_k(\mathbf{x})$ corresponding to the Absolute value penalty term has no critical points when

$$x_1^2 - x_2 < 2 \text{ or when } x_1^2 - x_2 > 2 \text{ for } k > 1.$$

Use this to compute the minimizer of $F_k(\mathbf{x})$ for $k > 1$.

3: Use the Penalty Function Method with Courant-Beltrami Penalty Term to minimize

$$f(x, y) = x^2 + y^2$$

subject to constraint $x + y \ge 1$.

4: Let $\varepsilon > 0$. Show that if a vector λ is a feasible for the dual (D) of a convex program (P), then λ is also feasible for (D^ε).

5: We know that if a convex program is superconsistent, then $MP = MD$. Show that the converse is not true. That is: find a convex program that is not superconsistent and yet $MP = MD$.
Hint: You can use the fact the for all linear programs $MP = MD$.