Math-484 Homework #5 (Geometric programming, KKT conditions)

Due 11am Mar 3.
Write your name on your solutions and indicate if you are a C14 (4 credit hour) student.

1: Suppose that 400 cubic yards of gravel must be ferried across a river. Suppose that the gravel is to be shipped in an open box of length t_1, width t_2, and height t_3. The sides and bottom of the box cost \$10 per square yard and the ends of the box cost \$20 per square yard. The box will have no salvage value and each round trip of the box on the ferry will cost 10 cents. What is the minimum total cost of transporting 400 cubic yards of gravel? *Hint: You can find this problem and its solution in the book where it is formulated as a geometric program. I encourage you to attempt to formulate it yourself a geometric problem before checking the book. You must use geometric programming and show your work for credit.*

2: State the dual (DGP) of the following (GP) and solve the (GP) using (DGP). Solving means, finding optimal $\mathbf{x}^* = (x_1, x_2)$ and value of the objective function.

$$(GP)\begin{cases}\text{Minimize} & (5^4)\frac{x_2^2}{x_1} + \frac{x_3}{5x_1x_2^2} + \frac{25x_1}{2} + \frac{1}{10x_1x_3^2}\\ \text{subject to} & x_1, x_2, x_3 > 0\end{cases}$$

You should verify your answers using `http://www.wolframalpha.com` or equivalent.

3: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

$$(P)\begin{cases}\text{Minimize} & f(x_1, x_2) = e^{-(x_1+x_2)}\\ \text{subject to} & e^{x_1} + e^{x_2} \leq 20\\ & x_1 \geq 0\end{cases}$$

4: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

$$\begin{cases}\text{Minimize} & f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 4x_2\\ \text{subject to} & x_1^2 - x_2 \leq 0\\ & x_1 + x_2 \leq 2\end{cases}$$

5: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

$$(P)\begin{cases}\text{Minimize} & -x_1 + x_2\\ \text{subject to} & x_1^2 + x_1 - x_2 - 2 \leq 0\\ & 11x_1 + 5x_2 - 6 \leq 0\end{cases}$$