Math-484 List of definitions and theorems

The following definitions and theorems are new for the final. The material on each of the 4 previous midterms will be covered on the final. That material is included in this document for convenience. As well as review the definitions and theorems listed here, please review the first four midterms and the 10 homework assignments to prepare for the final.

Definitions (New for Final):

- Jacobian Matrix of a function $g : \mathbb{R}^n \to \mathbb{R}^n$ *page 85*
- describe Newton's method for function minimization *page 88, 3.1.3*
- describe Steepest descent method *page 98, 3.2.1*

Theorems and statements (New for Final):

(*Try to not ignore assumptions - like that sometimes the function must be continuous etc.*)
(*Proofs are only for C14 (4 credit hour) students*)

- When Newton's method converges in one step? *Theorem 3.1.4*
- When is Newton's method guaranteed to do decreasing steps? *Theorem 3.1.5*
- What is the special property of the steps in Steepest descent method? *Theorem 3.2.3*
- When is the Steepest descent method really a descent method? *Theorem 3.2.5*
- What is a sufficient condition for the Steepest method to converge? *Theorem 3.2.6*
- State the conditions that a good descent method should satisfy. Write them formally as well as simple explanation in English. *(page 106,107), 4 credits also why they do that they do*
- State Wolfe's Theorem about existence about descent methods. *Theorem 3.3.1*

Definitions (Midterm 1):

- cosine of two vectors *page 6*
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *page 6*
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and r?) *page 6*
- interior D^0 of set $D \subseteq R^n$ *page 6, page 164*
- open set $D \subseteq \mathbb{R}^n$ *page 6*
- closed set $D \subseteq \mathbb{R}^n$ *page 7*
- compact set $D \subseteq \mathbb{R}^n$ *page 6*
- (global,local)(strict)minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- Hessian $Hf(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- quadratic form associated with a symmetric matrix A *page 12*
- (positive,negative)(semi)definite matrix *page 13*
- indefinite matrix *page 13*
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 23*
- Δ_k, the k^{th} principal minor of a matrix A *page 16*
- coercive functions *page 25*
- eigenvalues and eigenvectors of a matrix A *page 29*
- convex sets in $\mathbb{R}^n$ *page 38*
- closed and open half-spaces in $\mathbb{R}^n$ *page 40*
- convex combination of k vectors from $\mathbb{R}^n$ *page 41*
- convex hull of $D \subseteq \mathbb{R}^n$ *page 42*
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ *page 49*

Theorems and statements (for Midterm 1):

(*Try to not ignore assumptions - like that sometimes the function must be continuous etc.*)
(*Proofs are only for C14 (4 credit hour) students*)
- State Cauchy-Swartz inequality *(page 6)*
- Minimizers and maximizers of continuous function $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval *(Theorem 1.1.4)*
- local minimizers and the gradient *(Theorem 1.2.3)*
- Taylor's formula for $\mathbb{R}^n$ *(Theorem 1.2.4)*
- H_f and global minimizers and maximizers? *(Theorem 1.2.5/Theorem 1.2.9)*
- Principal minors of matrix A and there relation to positive(negative) (semi)definite matrices A *(Theorem 1.3.3)*
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices *(Theorem 1.5.1)*
- Hf and local minimizers and maximizers. *(Theorem 1.3.6, **with proof**)*
- coercive functions and minimization *(Theorem 1.4.4, **with proof**)*
- The convex hull of $D \subseteq \mathbb{R}^n$, $co(D)$, is the set of all convex combinations of vectors from D. ($D \subseteq \mathbb{R}^n$) *(Theorem 2.1.4)*
- convex function and continuity *(Theorem 2.3.1)*
- minimizers of convex functions *(Theorem 2.3.4 **with proof**)*
- inequality involving convex functions and convex combinations with the condition for equality *(Theorem 2.3.3)*
- maximizers of concave functions *(Theorem 2.3.4)*
- the relationship between convex function and the gradient *(Theorem 2.3.5)*
- critical points of convex function and minimization *(Theorem 2.3.5 + Corollary 2.3.6)*
- The relationship between the Hessian and convexity of a function (in $\mathbb{R}^n$) *(Theorem 2.3.7)*
- building convex function from other convex functions *(Theorem 2.3.10)*

Definitions (Midterm 2):

- posynomial *page 67*
- primal and dual geometric program *page 67,68*
- primal dual inequality for geometric programming *page 68*
- feasible point (or feasible vector) of a program (P) *page 169*
- feasible region of a program (P) *page 169*
- consistent program (P) *page 169*
- Slater point of a program (P) *page 169*
- superconsistent program (P) *page 169*
- solution of a program (P) *page 169*
- convex program (P) *page 169*
- Lagrangian $L(\mathbf{x}, \lambda)$ of a program (P)*page 182*
- complementary slackness conditions for a program (P) *page 184*
- saddle point of the Lagrangian of (P) *page 184*

Theorems and statements (Midterm 2):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)
(*Proofs are only for C14 (4 credit hour) students*)

- inequality involving convex functions and convex combinations with the condition for equality *(Theorem 2.3.3)*
- arithmetic-geometric mean inequality with the condition for equality *(Theorem 2.4.1 **with proof**)*
- Proof of the primal dual inequality for geometric programming *page 67,68* (**with proof**)
- Describe transition form unconstrained geometric program to its dual using A-G inequality (pages 67, 68). (**with proof**)
- For a geometric program, a solution to the primal implies a solution to the dual with equality in the primal-dual inequality *Theorem 2.5.2.*
- State Karush-Kuhn-Tucker Theorem (Saddle point version) *Theorem 5.2.13*
- State Karush-Kuhn-Tucker Theorem (Gradient form) *Theorem 5.2.14*

Definitions (Midterm 3):

- posynomial *page 67*
- hyperplane H in $\mathbb{R}^n$ *page 158*
- boundary point of $C \subset \mathbb{R}^n$ *page 158*
- closure $\overline{A}$ of $A \subset \mathbb{R}^n$ *page 163*
- epi(f) *page 167*
- subgradient of $f : \mathbb{R}^n \to \mathbb{R}$ *page 168*
- feasible point (or feasible vector) of a program (P) *page 169*
- feasible region of a program (P) *page 169*
- consistent program (P) *page 169*
- Slater point of a program (P) *page 169*
- superconsistent program (P) *page 169*
- solution of a program (P) *page 169*
- convex program (P) *page 169*
- linear program (LP) *page 173*
- Perturbation of P by z, $P(z)$ *page 174*
- $MP(z)$ *page 174*
- sensitivity vector of a program (P) *page 177*
- Lagrangian $L(\mathbf{x}, \lambda)$ of a program (P)*page 182*
- complementary slackness conditions for a program (P) *page 184*
- saddle point of the Lagrangian of (P) *page 184*
- general form of constrained geometric program (GP) and its dual (DGP) *page 193*
- dual of a convex program *page 200-201*
- $h(\lambda)$ *page 201*
- MD *page 201*
- feasible vector for the dual (DP) *page 201*
- consistent dual program (DP) *page 201*
- solution of the dual program (DP) *page 201*
- dual of a linear program (DLP) *page 202*

Theorems and statements (Midterm 3):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)

(*Proofs are only for C14 (4 credit hour) students*)

- If $C \subseteq \mathbb{R}^n$ is a convex set $y \in \mathbb{R}^n \setminus C$, then what is true if and only if $x^* \in C$ is the closest vector to y in C? *(Theorem 5.1.1)*
- What is the characterization of the closest vector of a convex set to a given vector using orthogonal complement? *(Theorem 5.1.2, **with proof**)*
- What is a sufficient condition for existence of a closest vector from a set C to a given vector $\mathbf{x}$? *(Theorem 5.1.3)*
- What is a sufficient condition for existence of a unique closest vector from a set C to a given vector $\mathbf{x}$? *Corollary 5.1.4*
- State basic separation theorem. *Theorem 5.1.5*
- State Support theorem. *Theorem 5.1.9*
- State *Theorem 5.1.10*
- What can you say about the function $MP(z)$ when (P) is superconsistent? *Theorem 5.2.6*
- Are there sufficient conditions for convex program (P) to have a sensitivity vector? *Theorem 5.2.8, **with proof***
- Can MP be computed from the sensitivity vector? *(Theorem 5.2.11), **with proof***
- State Karush-Kuhn-Tucker Theorem (Saddle point version) *Theorem 5.2.13*
- State Karush-Kuhn-Tucker Theorem (Gradient form) *Theorem 5.2.14*
- State Extended Arithmetic-Geometric Mean Inequality include the condition for equality! *Theorem 5.3.1, **with proof***
- What are sufficient condition for a constrained geometric program (GP) to have no duality gap? *Theorem 5.3.5*

Definitions (Midterm 4):

- duality gap *page 209*
- function $g^+(x)$ *page 215*
- absolute value penalty function *page 217*
- penalty parameter *page 217*
- Courant-Beltrami penalty function *page 219*
- generalized penalty function *page 223*
- generalized penalty objective function *page 223*
- Definition of $L^\epsilon(x, \lambda), P^\epsilon, MP^\epsilon, DP^\epsilon, MD^\epsilon$ *page 230*
- Definition of surface, normal space $N(\mathbf{x}^0)$ and tangent space $T(\mathbf{x}^0)$ to S at $\mathbf{x}^0$ (Theorem 7.1.1) *page 241*
- Definition of path in a surface S (Theorem 7.1.3) *page 243*
- Definition of a regular point (Theorem 7.1.3) *page 246*
- Definition of Lagrange multipliers and the Lagrange multiplier conditions *page 248*

Theorems and statements (Midterm 4):

- Description of first partial derivatives of $[g^+(x)]^2$ (Theorem 6.1.3) *page 219*
- Describe the penalty function method (6.2.1) *page 220*
- Description of the sequence of global minimizers of $P_k(x)$ and its convergent subsequence (Theorem 6.2.3) *page 221-222*
- No duality gap for a consistent convex program with a coercive objective function. (Theorem 6.3.1) *page 227*
- How to modify any convex function to a coercive one? (why it is coercive?) (with proof) *page 229*
- The relations between $P^\epsilon, MP^\epsilon, DP^\epsilon, MD^\epsilon$? (Theorem 6.3.2) *page 230*
- Relationship between $MP = \inf_{\varepsilon>0}\{MD^\varepsilon\}$? (with proof) (Theorem 6.3.4) *page 232*
- What can you say when (P) is superconsistent and $MP > -\infty$? (Theorem 6.3.5) *page 232-233*
- What is the relationship between vectors in $T(\mathbf{x}^*)$ and paths in S. *Theorem 7.1.5 page 244*
- What can you say about a local minimizer of a program that is also a regular point. *Theorem 7.2.1 page 244*