Math-484 List of definitions and theorems

The first midterm will cover the material covered on the first three homework assignments. You should review the homework assignments to prepare for the exam is to review. You should also know the following definitions and theorems.

Definitions (Midterm 1):

- cosine of two vectors *page 6*
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *page 6*
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and r?) *page 6*
- interior D^0 of set $D \subseteq R^n$ *page 6, page 164*
- open set $D \subseteq \mathbb{R}^n$ *page 6*
- closed set $D \subseteq \mathbb{R}^n$ *page 7*
- compact set $D \subseteq \mathbb{R}^n$ *page 6*
- (global,local)(strict)minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- Hessian $Hf(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- quadratic form associated with a symmetric matrix A *page 12*
- (positive,negative)(semi)definite matrix *page 13*
- indefinite matrix *page 13*
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 23*
- Δ_k, the k^{th} principal minor of a matrix A *page 16*
- coercive functions *page 25*
- eigenvalues and eigenvectors of a matrix A *page 29*
- convex sets in $\mathbb{R}^n$ *page 38*
- closed and open half-spaces in $\mathbb{R}^n$ *page 40*
- convex combination of k vectors from $\mathbb{R}^n$ *page 41*
- convex hull of $D \subseteq \mathbb{R}^n$ *page 42*
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ *page 49*

Theorems and statements (for Midterm 1):
(*Try to not ignore assumptions - like that sometimes the function must be continuous etc.*)
(*Proofs are only for C14 (4 credit hour) students*)
- State Cauchy-Swartz inequality *(page 6)*
- Minimizers and maximizers of continuous function $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval *(Theorem 1.1.4)*
- local minimizers and the gradient *(Theorem 1.2.3)*
- Taylor's formula for $\mathbb{R}^n$ *(Theorem 1.2.4)*
- H_f and global minimizers and maximizers? *(Theorem 1.2.5/Theorem 1.2.9)*
- Principal minors of matrix A and there relation to positive(negative) (semi)definite matrices A *(Theorem 1.3.3)*
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices *(Theorem 1.5.1)*
- Hf and local minimizers and maximizers. *(Theorem 1.3.6, **with proof**)*
- coercive functions and minimization *(Theorem 1.4.4, **with proof**)*
- The convex hull of $D \subseteq \mathbb{R}^n$, $co(D)$, is the set of all convex combinations of vectors from D. ($D \subseteq \mathbb{R}^n$) *(Theorem 2.1.4)*
- convex function and continuity *(Theorem 2.3.1)*
- minimizers of convex functions *(Theorem 2.3.4 **with proof**)*
- inequality involving convex functions and convex combinations with the condition for equality *(Theorem 2.3.3)*
- maximizers of concave functions *(Theorem 2.3.4)*
- the relationship between convex function and the gradient *(Theorem 2.3.5)*
- critical points of convex function and minimization *(Theorem 2.3.5 + Corollary 2.3.6)*
- The relationship between the Hessian and convexity of a function (in $\mathbb{R}^n$) *(Theorem 2.3.7)*
- building convex function from other convex functions *(Theorem 2.3.10)*