Saturday 30$^{\text{th}}$ April, 2016 10:34

Final topics

Only the E14 students need to know the proofs of the theorem marked with "(with proof)". Students in both E13 and E14 sections are required to know and understand the statement of the theorems listed.

A good way to study for the final is by reviewing the four midterms (midterm 4 will be passed back on Monday 5/2 with solutions). Also, the ten homework assignments should be reviewed. Key definitions and the theorems listed below should also be reviewed.

(1) Softened ellipsoid method (know the basic algorithm and how it is related to solving a linear program)
(2) size of a matrix/rational number/integer, e.g. for i an integer $\langle i\rangle = \lceil \log_2(i+1)\rceil + 1$.
(3) machine scheduling - now how to convert an optimal solution of $LPR(T)$ to a schedule
(4) Min-cost flow
(5) incremenatal weighted flow network $N'(f)$
(6) Floyd-Warshall
(7) Primal dual for shortest path (Section 5.4)
(8) Theorem 13.3 (with proof)
(9) Revised simplex
(10) 2-phase simplex and 2-phase revised simplex
(11) Farkas Lemma (Theorem 3.5)
(12) Matrix games, pure strategy, mixed strategy, minimax theorem (with proof), stochastic vertex, Alice, Bob, optimal strategy, value of game, solving a matrix game using linear programming (see handout on website)
(13) A circulation is the sum of flows on cycles, and a (s,t)-flow is the sum of flows on (s,t)-path, cycles and (t,s)-paths
(14) revised simplex method and two phase revised simplex (section 4.1)
(15) Max-flow via revised simplex (Section 4.3)
(16) Value of flow f: $|f|$
(17) Theorem 5.1 - with proof
(18) Theorem 5.3 - with proof
(19) Ford-Fulkerson algorithm for max-flow
(20) definition of an $s-t$-cut (section 6.1)
(21) Theorem 6.1 - with proof
(22) max-flow = min-cut
(23) f-augmenting (s,t)-path
(24) Integer programming and satisfiability
(25) Theorem 13.3 (with proof)
(26) The incidence matrix of bipartite graphs and directed graphs are TUM (Theorem 13.3 Corollary)
(27) (Koening's theorem) Min vertex cover = max matching in bipartite graphs - via integer programming (with proof)
(28) Relaxation of a integer linear program
(29) The basic feasible solutions the LP in standard or canonical form are integer if the matrix is totally unimodular
(30) Totally unimodular matrices - definition
(31) Weak Duality - If x is feasible for P and π is feasible for its dual D, then $\pi^T b \leq c^T x$.
(32) Finding the dual of a linear program in general form (Definition 3.1)
(33) Strong Duality (Theorem 3.1) (with proof - can assume P in standard form)
(34) Dual of the dual is primal (Theorem 3.2) (with proof)
(35) Theorem 3.3 in book
(36) Complementary slackness (section 3.4) (with proof)
(37) Dual simplex method (Section 3.6)
(38) Using the final tableau to solve a slightly modified LP.

(39) Farkas Lemma (Theorem 3.5)
(40) Matrix games, pure strategy, mixed strategy, minimax theorem (with proof), stochastic vertex, Alice, Bob, optimal strategy, value of game, solving a matrix game using linear programming (see handout on website)
(41) Simplex method and two phase simplex method (see examples on website)
(42) $\bar{c}_j$ relative cost of column j, and the relative cost vector $\bar{c}$.
(43) Lexicographic simplex - know the row selection rule pivot rules
(44) Bland's rule - know the column and row selection rules and how to apply them
(45) Fact that lexicographic simplex and Bland's rule do not cycle
(46) Algebraic Theorem about pre-multiplication matrix and updated Tableau $\widetilde{T}$ (with proof) (see handout on website).
(47) Definition: feasible solution
(48) Definition: object function
(49) Definition: optimal solution, optimum
(50) Definition: infeasible LP
(51) Definition: unbounded LP
(52) Definition: basis
(53) Definition: basic feasible solution
(54) Definition: degenerate basic feasible solution
(55) Definition: feasible basis
(56) Definition: Standard/Canonical/General form and how to transform to each form.
(57) Definition: lex positive, lex negative, lex zero
(58) Solving a 2d LP graphically
(59) Prop 1 - A feasible basis B corresponds to exactly one basic feasible solution x and x is defined by $x_B = A_B^{-1}b$ and $x_j = 0$ for all j not in B.
(60) Prop 2 - Let x be a feasible solution to an LP in standard form. x is a basic feasible solution if and only if the columns of A_k are linearly independent where $K = \{j \in [n] : x_j > 0\}$.
(61) Fundamental theorem:
Suppose LP is a linear program in standard form.
- If the LP has a feasible solution, then it has a basic feasible solution.
- If the LP has no optimal solution, then the LP is infeasible or unbounded.
- If the LP has an optimal solution, then it has a optimal basic feasible solution.