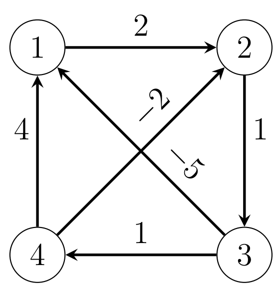
Math 482 **HW10** Section: ________ Name: ______________________________

Due Friday, April 29, 2016

Students in section E13 (three credit hours) need to solve any four (question two counts as two problems) of the following five problems. Students in section E14 (four credit hours) must solve all of the problems.

1. Use the Floyd-Warshall algorithm to find a negative cost cycle in the following directed graph. Your answer should include all of the matrices D^j and E^j that you constructed and should describe how you used the matrices to detect and reconstruct a negative cost cycle.



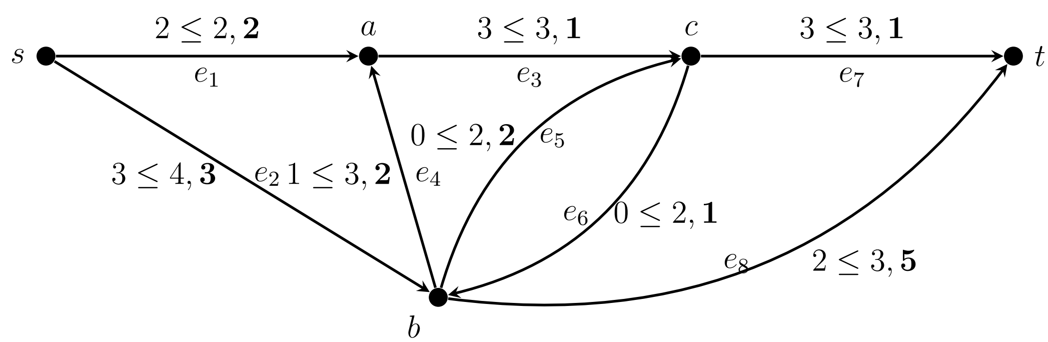
2. THIS PROBLEM IS COUNTED AS TWO PROBLEMS.
Let N be the network consisting of the directed graph with edges $E = [sa, sb, ac, ba, bc, cb, ct, bt]$, capacities $b = [2, 4, 3, 3, 2, 2, 3, 3]^T$, costs $c^T = [2, 3, 1, 2, 2, 1, 1, 5]$ and let $f = [2, 3, 3, 1, 0, 0, 3, 2]^T$ be a flow. This network and flow is drawn below. Our aim is to construct a min-cost flow with value 5.

   (a) Compute the cost of the given flow.

   (b) Draw the incremental weighted flow network $N'(f)$ and find a negative cost f^r cycle in $N'(f)$ (you do not need to use the Floyd-Warshall algorithm for this step).

   (c) Then find the maximum θ such that $f^* = f + \theta f^r$ is a feasible flow.

   (d) Determine if the flow $f^* = f + \theta f^r$ is a minimum cost flow with value 5.



3. Assume that the following modified version Farkas' lemma is true:
For any $A \in Q^{m \times n}$ and $b \in \mathbb{Q}^n$, there exists $\omega \in \mathbb{N}$ and $y \in \mathbb{Q}^m$ exactly one of the following two statements hold:

   - there exists $x \in \mathbb{Q}^n$ such that $Ax \leq b$, or
   - there exists $y \in \mathbb{Q}^m$ such that $y^T A = 0$, $y \geq 0$, $y^T b = -1$ and $|y_i| < 2^\omega$ for every $i \in [m]$.

   Let A, b, ω and y be as in the statement of the modified version of Farkas' Lemma above. Prove that if $\eta = 1/(2^\omega m)$ and there exists $\tilde{x} \in \mathbb{Q}^n$ such that $A\tilde{x} \leq b + \eta\mathbf{1}$, then there exists a solution to $Ax \leq b$.

4. Let $b \in \mathbb{Q}^m$ and $A \in \mathbb{Q}^{m \times n}$ such that $|a_{i,j}| \leq 2^\omega$ for every $i \in [m]$ and $j \in [n]$. Prove that for any $\eta > 0$ and any $x \in \mathbb{Q}^n$ such that $Ax \leq b$, if $\varepsilon = \eta/(2^\omega n)$ and $y \in \mathbb{R}^n$ such that $\|y - x\| \leq \varepsilon$, then $Ay \leq b + \eta\mathbf{1}$.
Hint: You can freely use that fact that if $\|y - x\| \leq \varepsilon$, then $|y_j - x_j| \leq \varepsilon$ for every $j \in [n]$