Math 482 **HW7** Section: ____________ Name: ______________________________

Due Friday, April 1, 2016

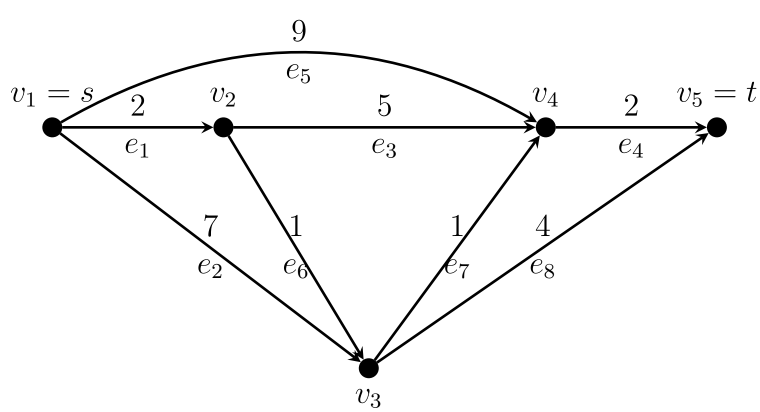
Students in section E13 (three credit hours) need to solve any four of the following five problems. Students in section E14 (four credit hours) must solve all five problems.

1. Consider the following integer program P:

$$
\begin{array}{rccll}
 & z = x_1 & \rightarrow & \min & \\
\text{subject to} & 3x_1 & -100x_2 & \geq & 1 \\
 & 3x_1 & -101x_2 & \leq & 1 \\
 & x_1, & x_2 & \geq & 0 \\
 & x_1, & x_2 & & \text{integer}
\end{array}
$$

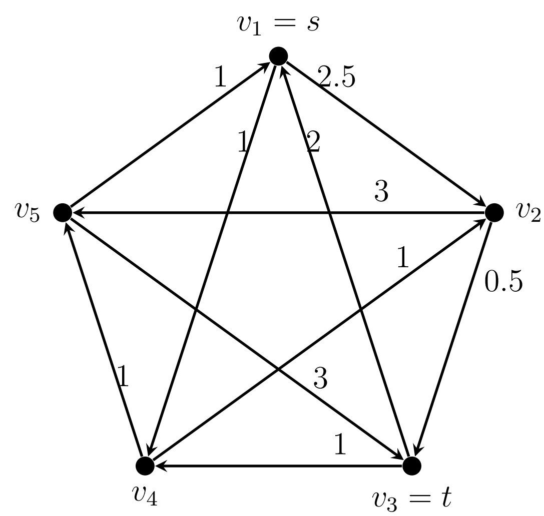
   Solve the linear programming relaxation of P, obtaining an optimal solution x^* with cost z^* (You can solve the linear programming relaxation in any manner that you wish). Obtain an integer vector x from x^* by rounding each component to the nearest integer. Is x an optimal solution to the integer program P? If it is not, find an optimal solution to the integer program P.
   Hint: Since the objective function of the integer linear program is just x_1, you can find an optimal solution by checking if there is a feasible solution in which $x_1 = 0$, then checking if there is a feasible soluition in which $x_1 = 1$, etc.

2. Interpreting the numbers on edges as edge lengths, solve the shortest (s, t)-path problem for the graph drawn below using the simplex method with Bland's pivot rules with the initial basis $\{e_1, e_2, e_3, e_4\}$.
   Hint: this is similar to example 3.7 in the book



3. State the dual to the shortest path problem above and use your solution to problem 2 and complementary slackness to give its solution.

4. Let G be the network with the flow drawn below with $s = v_1$ and $t = v_3$. Write the flow as a sum combination of positive flows along cycles and (s,t)-paths.



5. Use the revised simplex method to find an optimal solution to the problem

$$\text{Minimize } z = x_1 + x_2 + x_3$$

subject to

$$\begin{cases} x_1 + x_4 - 2x_6 &= 5, \\ x_2 + 2x_4 - 3x_5 + x_6 &= 3, \\ x_3 + 2x_4 - 3x_5 + 6x_6 &= 5, \\ x_1, \ldots, x_6 &\geq 0. \end{cases}$$