**Math 482** **HW6** Section: __________ Name: ______________________________

Due Friday, March 11, 2016

Students in section E13 (three credit hours) need to solve any four of the following five problems. Students in section E14 (four credit hours) must solve all five problems.

1. Solve the game with the payoff matrix

$$\begin{pmatrix} 3 & 5 & 3 & -2 & 0 \\ 3 & 7 & 3 & -1 & 1 \\ 2 & -4 & 1 & 3 & 4 \\ 1 & -5 & 1 & 3 & 0 \end{pmatrix}$$

   Here both players are allowed to use mixed strategies. You must use dominance relation to reduce the matrix to a $2 \times 3$ matrix. You can either use the simplex method or a linear program solver to get a optimal mixed strategy for the row player Alice, but you should express the answer exactly, i.e. as fractions not decimals. Your answer should be with respect to the original $4 \times 5$ matrix.

2. Recall the game Morra discussed in class: Each player plays either 1 or 2 and then, at the same time, they guess what the other player has played. If exactly one of the two players guesses correctly, then that player wins the *total* amount played. For example, suppose Alice plays 2 and Bob plays 1. If Alices guesses that Bob played 1 and Bob guesses that Alice played 2, then no money changes hands, and if Alices guesses that Bob played 2 and Bob guesses that Alice played 2, then Bob when \$3 from Alice.

   Consider the following modification to the game, suppose that, after playing, Bob always guesses first, so Alice always knows what Bob has quessed before she guesses. This changes the game as Alice has 4 new additional pure strategies: she can play 1 and repeat Bob's guess, play 1 and guess differently than Bob, play 2 and repeat Bob's guess, or play 2 and guess differently than Bob. So Alice has 8 pure strategies in this game, while Bob still only has 4 pure strategies. Find the matrix that corresponds to this modified game and find a worst case optimal mixed strategy for Alice and find the value of the game.

   *You can and should use a computer to solve this linear program, but please write the linear program you are solving clearly and write your answer exactly, i.e. as a fraction*

3. Let $G = (V, E)$ be an undirected graph. A set $U \subseteq V$ is an independent set in $G$, if there does not exist an edge $\{v_i, v_j\} \in E$ such that both $v_i \in U$ and $v_j \in U$. Formulate the problem of finding a maximum size independent set in $G$ as an integer linear program.

4. Prove that if $A$ is TUM, then the matrices $A^T$, $-A$ and $(A|A)$ are all TUM.

5. Show that every matrix with entries $-1, 0$ and $1$ such that at most one row has more than one non-zero entry is totally unimodular. Give an example of a totally unimodular matrix with at least three non-zero elements in every column and in every row.