Math 482 **HW5** Section: ____________ Name: ______________________________________

Due Friday, March 4, 2016

Students in section E13 (three credit hours) need to solve any four of the following five problems. Students in section E14 (four credit hours) must solve all five problems.

Note: Question 2, 3 and 4 involve a fair amount of computation. You are encouraged to use a computer algebra system to check that the row operations are done correctly. You must write the tableau after each pivot steps.

1. Prove the theorem due to P. Gordan (1873) that the system $\mathbf{Ax} < \mathbf{0}$ is unsolvable if and only if the system $\mathbf{y^T A} = \mathbf{0}, \quad \mathbf{y} \geq \mathbf{0}, \quad \mathbf{y} \neq \mathbf{0}$ is solvable. (Hint: In order to apply duality theorems, replace the system $\mathbf{Ax} < \mathbf{0}$ of strict inequalities by the system $\mathbf{Ax} \leq -\mathbf{1}$ of nonstrict inequalities. Prove that the new system is solvable if and only if $\mathbf{Ax} < \mathbf{0}$ is solvable).

2. Use the dual simplex method to find an optimal solution to the problem

$$\text{Minimize } z = 7x_1 + x_2 + 3x_3 + x_4$$

subject to

$$\begin{cases} 2x_1 - 3x_2 - x_3 + x_4 & \geq & 8, \\ 6x_1 + x_2 + 2x_3 - 2x_4 & \geq & 12, \\ -x_1 + x_2 + x_3 + x_4 & \geq & 10, \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{cases}$$

3. The additional constrains

$$\begin{array}{rcl} x_1 + 5x_2 + x_3 + 7x_4 & \leq & 50, \\ 3x_1 + 2x_2 - 2x_3 - x_4 & \leq & 20 \end{array}$$

are added to those of Problem 2. Solve the new problem starting from the optimal tableau for Problem 2. After excluding the variables that are in the basis from the rows that you will add, do one pivot step. If the tableau after this step gives an optimal solution to the problem, write this optimal solution and the value of the objective function at this solution.

4. Suppose the objective function in problem 2 is changed to

$$\text{Minimize } z = 7x_1 - x_2 + 3x_3 + x_4$$

Solve the new problem starting from the optimal tableau for Problem 2.

5. Let the matrix define a game and let Alice be the player whose pure strategies are represented by the rows of the matrix and Bob be the player whose pure strategies are represented by the columns of the matrix. What is the optimal pure strategy for Alice and what is expected payout given that choice? In other words, if Alice must play the same pure strategy in every turn of the game, what pure strategy should she play and what is the expected payout assuming Bob plays optimally? Determine the same information for Bob.

$$\begin{pmatrix} 4 & 8 & 1 & 2 & 6 & 7 \\ 8 & 2 & 3 & 5 & 9 & 2 \\ 1 & 8 & 1 & 7 & 3 & 3 \\ 6 & 5 & 6 & 8 & 6 & 3 \\ 2 & 6 & 2 & 4 & 7 & 1 \\ 3 & 9 & 1 & 5 & 2 & 6 \end{pmatrix}.$$