Math 482 **HW4** Name: ____________________

Due Friday, February 19, 2016

Students in section D13 (three credit hours) need to solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Suppose that there exists x_0 and y such that x_0 is feasible for the linear program P

$$\begin{array}{rrcl} \min & c^T x & & \\ \text{subject to} & Ax & = & b \\ & x & \geq & 0 \end{array}$$

   and y satisfies

$$c^T y < 0$$
$$Ay = 0$$
$$y \geq 0.$$

   Prove that P is unbounded. *(Hint: You can prove this directly. You do not need anything from Chapter 3 (Duality) to solve this problem)*

2. State the dual to the following problem:

$$z = 3x_1 - x_2 - 2x_3 \quad \longrightarrow \quad \min$$

   with respect to

$$\begin{cases} 6x_1 & -2x_2 & +3x_3 & & +2x_5 & \geq & -7, \\ -2x_1 & +3x_2 & & -2x_4 & & \leq & 6, \\ 2x_1 & +x_2 & -4x_3 & +x_4 & & = & 6, \\ & & & x_1, & x_2 & x_4 \geq & 0 \end{cases}$$

3. The Transportation Problem (known also as the Hitchcock problem) is as follows. There are m sources of some commodity, each with a supply of a_i units, $i = 1, \ldots, m$ and n terminals, each of which has a demand of b_j units, $j = 1, \ldots, n$. The cost of sending a unit from source i to terminal j is c_{ij} and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. We want to find a cheapest way to satisfy all demands. State this problem as an LP, and state the dual to this problem. *(Hint: The answer can be expressed in a compact form.)*

4. Use the complementary slackness condition to check whether the vector $[3, -1, 0, 2]^T$ is an optimal solution to the problem

$$\text{Maximize } z = 6x_1 + x_2 - x_3 - x_4$$

   subject to

$$\begin{cases} x_1 & +2x_2 & +x_3 & +x_4 & \leq & 5, \\ 3x_1 & +x_2 & -x_3 & & \leq & 8, \\ & x_2 & +x_3 & +x_4 & = & 1, \\ & & x_3, & x_4 & \geq & 0. \end{cases}$$

5. Prove that if P is an LP in standard form, P has an optimal solution, and P has no degenerate optimal solutions, then there is a unique optimal solution to the dual of P. (Hint: Use the complementary slackness condition and the fact that if an LP in standard form has an optimal solution, then it has an optimal basic feasible solution)