

Due Friday, September 11, 2015

Students in section D13 (three credit hours) need to solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Find all of the basic feasible solutions of the following LP, the value of the objective function at each of the basic feasible solutions and the associated basis for each basic feasible solution. From this, determine an optimal solution and optimal value for the LP. **Do not use the simplex method on this problem**

$$z = 4x_1 + 2x_2 + x_3 \longrightarrow \min$$

subject to

$$\begin{cases} 2x_1 + 4x_2 + 2x_3 = 10 \\ x_1 + 2x_2 + 3x_3 = 6 \\ x_1, x_2, x_3 \geq 0. \end{cases}$$

2. A farmer has 100 acres of land which he has decided to make part arable and part grass. Part of the land may also lie fallow. He can make an annual profit of \$150/acre from arable land and \$100/acre from grass. Each year arable land requires 25 hours work per acre, and grass requires 10 hours work per acre. The farmer does not want to work more than 2000 hours in any year. How should he divide his land between arable and grass so as to maximize his annual profit? You should formulate the problem as a linear program and write down this formulation as part of your solution.
3. Solve the following LP using the simplex method. You should find an optimal feasible solution and the value of the objective function at (or cost of) the solution. Draw in the plane $\mathbf{0x_1x_2}$ the feasible region for the problem. In this picture, mark all of the points corresponding to the basic solutions arising at the steps of the simplex method. (You must use the simplex method to receive points for this problem).

$$\text{Maximize } z = 5x_1 + 3x_2$$

subject to

$$\begin{cases} x_1 + x_2 \leq 5, \\ x_1 - x_2 \leq 3, \\ x_1, x_2 \geq 0. \end{cases}$$

4. Solve the LP represented by the tableau below using the simplex method.

		x_1	x_2	x_3	x_4	x_5
$-z$	-6	(2)	0	-1	0	3
\mathbf{x}_1	3	1	0	-1	1	4
\mathbf{x}_2	2	0	1	0	1/2	2

Be careful. Note that $a_{0,1}$ (the circled entry) is not equal to 0.

5. Prove that if variable x_s is moved out of the basis of a linear program at some step of the simplex method, then at the next step it will NOT be moved back into the basis.