Cycling example (2.7 from book)

- This is an example 2.7 from the book and is an example of cycling.
- ► The first tableau *T*₁ will appear again as tableau *T*₇ when we use the following natural pivot rules.
- ▶ Select the pivot column *s* so that $a_{0,s} = \overline{c}_s \leq \overline{c}_j = a_{0,j}$ for all $j \in [n]$ (In this example, this always gives a unique choice)
- In the case of ties when selecting the pivot row, select the row so that the smallest index leaves the basis (this rule is the same as Bland's rule)

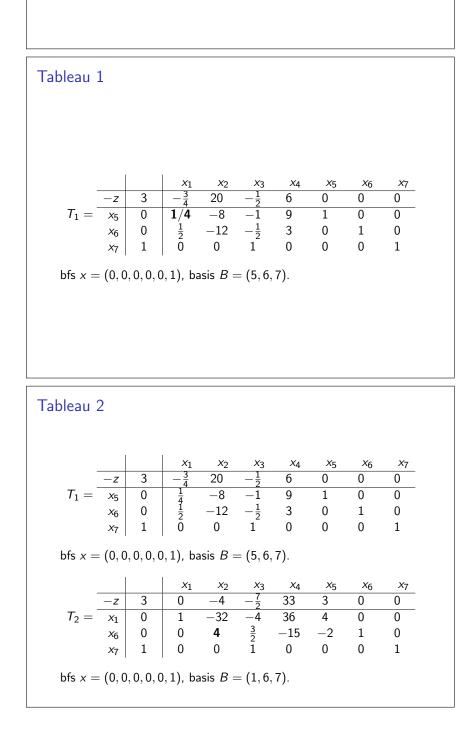


Tableau 3 x_2 X3 *x*₄ X_5 *x*6 *X*7 20 3 6 0 0 0 - Z $T_1 =$ 0 -8 9 1 0 0 $^{-1}$ *x*5 1412 0 0 -12 $-rac{1}{2}$ 1 3 0 1 0 *x*6 1 0 0 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). *X*4 X3 *X*5 *X*6 *X*7 x_1 X_2 3 0 0 -2 18 1 0 1 -*z* 0 0 0 1 8 -84 -12 8 1 4 0 0 1 $T_3 =$ x_1 $-\frac{15}{4}$ 0 $-\frac{1}{2}$ 0 0 $\frac{3}{8}$ 0 *x*₂ 1 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (1, 2, 7). Tableau 4 *x*₄ *X*5 *x*6 *X*7 6 0 20 3 0 0 0 -19 0 $T_1 =$ -8 1 *X*5 14 12 0 $-\frac{1}{2}$ 0 -123 0 1 0 *x*6 1 0 1^{-} 0 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). XΔ X7 0 0 3 0 0 -3 -2 3 -*Z* $-\frac{21}{2}$ $\frac{3}{16}$ 21/2 $\frac{1}{-\frac{1}{8}}$ -1 $-\frac{3}{2}$ $\frac{1}{16}$ $\frac{3}{2}$ 0 $\frac{1}{8}\frac{3}{64}$ 0 1 0 1 0 0 $T_4 =$ X3 0 0 *x*₂ 1 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (3, 2, 7). Tableau 5 *x*4 *X*7 X_2 X5 20 0 3 6 0 0 0 $-\overline{1}$ 9 0 0 $T_1 =$ -8 1 *X*5 14 12 0 $-\frac{1}{2}$ 1 0 -123 0 1 0 *x*6 1 0 0 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). *X*4 X_5 X7 X_2 X_3 X_6 X_1 16 0 3 0 0 -1 1 **2** ¹/₃ -2 0 56 1 0 0 $T_5 =$ -6 X3 $\frac{2}{\frac{1}{4}}$ $\frac{16}{3}$ -56 $-\frac{2}{3}$ 6 0 0 1 0 *x*4 0 0 1 1 *X*7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (3, 4, 7).

Tableau 6 *x*4 X7 X_2 *X*5 *x*6 20 3 6 0 0 0 $T_1 =$ 0 -8 9 1 0 0 *X*5 $^{-1}$ 1412 0 0 -12 $-\frac{1}{2}$ 3 0 1 0 *x*6 1 0 1 0 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). X_1 X2 *X*4 Xъ X_6 *X*7 3 44 0 0 -2 0 ٠z 0 28 0 1 -3 0 $\frac{1}{2}$ $-\frac{1}{6}$ $T_6 =$ *X*5 0 $\frac{1}{6}$ $\frac{1}{3}$ 1 0 -4 0 *x*4 0 1 1 0 0 *X*7 1 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 4, 7). Tableau 7 same as Tableau 1! *x*₂ X_4 *X*5 *x*6 X7 3 0 20 6 0 0 0 $T_1 =$ *x*5 1 4 1 2 0 -8 $-\overline{1}$ 9 1 0 0 0 -12 $-\frac{1}{2}$ 3 0 1 0 *x*6 1 0 0 0 1 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). x_6 X_4 X_5 *X*7 Xo 3 20 0 0 6 0 0 0 -8 9 0 $T_{7} =$ 1 *X*5 $^{-1}$ 1412 0 0 $-\frac{1}{2}$ -123 0 1 0 *x*6 1 0 1 0 0 0 1 X7 bfs x = (0, 0, 0, 0, 0, 1), basis B = (5, 6, 7). Example with Lexicographic simplex ▶ We start with the same tableau as example 2.7 from the book, but this time we following the lexicographic simplex method. > That is, in the case of ties when selecting the pivot row, we select the row that is smallest in lexicographic order.

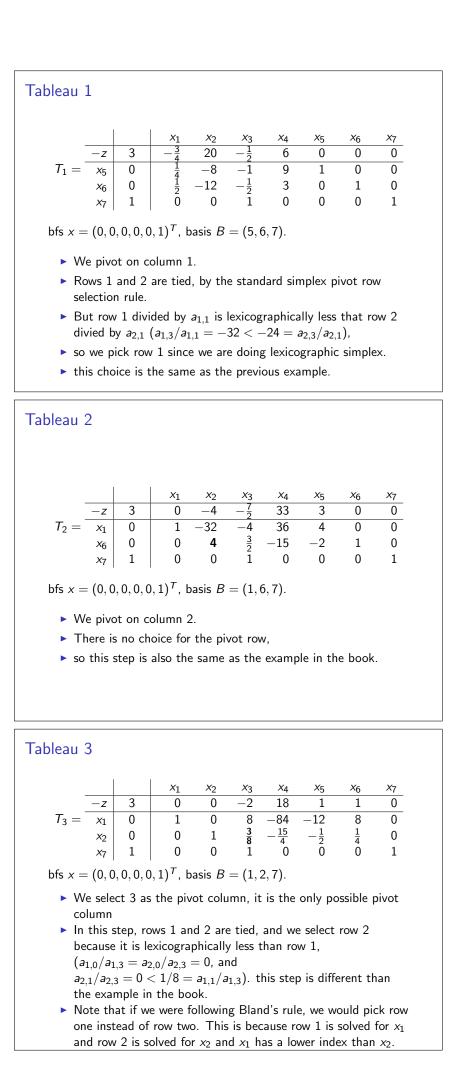


Tableau 4

$$T_4 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & 0 & 16/3 & 0 & -2 & -5/3 & 7/3 & 0 \\ \hline x_1 & 0 & 1 & -64/3 & 0 & -4 & -4/3 & 8/3 & 0 \\ \hline x_3 & 0 & 0 & 8/3 & 1 & -10 & -4/3 & 2/3 & 0 \\ \hline x_7 & 1 & 0 & -8/3 & 0 & 10 & \mathbf{4/3} & -2/3 & 1 \end{vmatrix}$$

bfs $x = (0, 0, 0, 0, 0, 1)^T$, basis B = (3, 2, 7).

- ▶ We pick column 5 as our pivot column, but *x*₄ would also be a valid choice.
- Once we select column 5 as our pivot column, we must pivot on row 3

Tableau 5

			x_1	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>x</i> 5	<i>x</i> ₆	<i>x</i> 7
	- <i>z</i>	17/4	0	2	0	21/2	0	3/2	5/4
$T_5 =$	<i>x</i> ₁	1	1	-24	0	6	0	2	1
	<i>x</i> 3	1	0	0	1	0	0	0	1
	<i>X</i> 5	3/4	0	-2	0	15/2	1	$2 \\ 0 \\ -1/2$	3/4

bfs $x = (1, 0, 1, 0, 3/4, 0)^T$, basis B = (1, 3, 5).

This is the optimal solution, because the entries in the top row (columns 1 thru 7) are non-negative.

Quiz

 Find the pivot entry using Bland's rule and lexicographic simplex

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	<i>x</i> 6	<i>X</i> 7
-z	8	2	0	0	2	-3	0	0
<i>X</i> 7	2	2	0	0	3	4	0	1
<i>x</i> 6	6	10	0	0	4	12	1	0
<i>x</i> 3	4	5	0	1	2	8	0	0
<i>x</i> ₂	2	0	1	0	4	3	0	0

• Using Bland's rule we select column x_5 and row 3

• Using lexicographic simplex we select column x_5 and row 1