Math 412 HW7

Due Friday, April 1, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

- 1. Let G be a bipartite graph. Prove that if $\delta(G) > n(G)/4$, then $\kappa'(G) = \delta(G)$.
- **2**. Prove that $\kappa(G) = \delta(G)$ when G is a graph with girth 5 and diameter 2.
- 3. Prove that a connected graph is k-edge-connected if and only if each of its blocks is k-edge-connected.
- 4. Use Menger's Theorem $(\kappa(x,y) = \lambda(x,y))$ to prove the König-Egeváry Theorem.
- **5**. For $k \geq 2$, prove that every k-connected graph with at least 2k vertices contains a cycle of length at least 2k.
- **6.** For $k \geq 2$, prove that a graph G with at least k+1 vertices is k-connected if and only if whenever $T \subseteq S \subseteq V(G)$ with |S| = k and |T| = 2, there is a cycle in G that contains T and avoids S T.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 4.1: # 1, 2, 3, 5, 7.

Section 4.2: # 1, 2, 3, 4, 6.

OTHER INTERESTING PROBLEMS:

Section 4.1: # 10, 12, 18, 19, 31. Section 4.2: # 8, 9, 11, 12, 14, 20, 26.