Math 412   **HW5**

Due Friday, March 4, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. Prove that a d-regular simple graph G has a decomposition into copies of $K_{1,d}$ if and only if G is bipartite.

2. Count the number of graphs G such that $V(G) = \{0, \ldots, n-1\}$, $e(G) = n-1$ and $f(v) = v$ is a graceful labeling of G. Here we count two graphs as different if their edge set is different.

3. In a weighted complete graph, iteratively select the edge of least weight such that the edges select so far form a disjoint union of paths. After $n-1$ steps, the results is a spanning path. Either prove that this algorithm always gives a minimum weight spanning path or give an example where it fails to give a minimum weight spanning path.

4. **Prim's Algorithm** grows a spanning tree from a given vertex of a connected weighted graph, iteratively adding the cheapest edge from a vertex already reached to a vertex not yet reached, finishing when all the vertices of G have been reached. (Ties are broken arbitrarily.) Prove that Prim's Algorithm produces a minimum weight spanning tree.

5. Let C be a cycle in a connected weighted graph in G. Let e be an edge of maximum weight on C. Prove that there is a minimum spanning tree not containing e. Use this to prove that iteratively deleting a heaviest non cut-edge until no such edge exists produces a minimum weight spanning tree.

6. Two people play a game on a graph G, alternately choosing distinct vertices. The first player starts by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice (of the other player). Thus together they follow a path. The last player able to move wins.

Prove that the second player has a winning strategy if G has a perfect matching and the first player has a winning strategy if G does not have a perfect matching.

————————————————————————

Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section 2.3: # 2, 3.
Section 3.1: # 1, 2, 3, 4, 6.
Do not write these up!

OTHER INTERESTING PROBLEMS:
Section 2.2: # 23
Section 3.1: # 8, 9, 10, 21, 24, 25, 30.
Do not write these up!