Math 412 **HW4**

Due Friday, February 26, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. Let T, T' be spanning trees of a connected graph G. For any $e \in E(T) - E(T')$ prove that there exists $e' \in E(T') - E(T)$ such that both $T' + e - e'$ and $T - e + e'$ are spanning trees of G.

2. Let T be a tree on with an even number of vertices. Prove that T has exactly one spanning subgraph such that every vertex has odd degree.

3. Given $x \in V(G)$, let $s(x) = \sum_{v \in V(G)} d(x, v)$. The *barycenter* of G is the set its vertices at which $s(x)$ is minimized.

a) Prove that the barycenter of a tree is a single vertex or two adjacent vertices. (Hint: Study $s(u) - s(v)$ when $uv \in E(G)$.)

b) Give an example of a tree in which the distance between the center and the barycenter is at least 3.

4. Using the Prüfer correspondence, for $n \geq 6$, count the number of trees with vertex set $[n]$ that have maximum degree 3 and exactly four leaves.

5. Let x be a vertex in a graph G, and suppose that $k = \epsilon(x) > \operatorname{rad} G$. (a) Prove that if G is a tree, then x has a neighbor with eccentricity $k - 1$.
(b) Show that part (a) does not hold for all graph by constructing, for every even r that is at least 4, a graph with radius r in which x has eccentricity $r + 2$ and has no neighbor of eccentricity $r + 1$. (Hint: For the construction, use a connected graph with exactly one cycle.)

6. Calculate $\tau(K_{2,4})$ using the Matrix Tree Theorem. How many **non-isomorphic** spanning trees are contained in $K_{2,4}$? (As always, you must prove that your answer is correct).

Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section Section 2.2: # 1, 2, 3. Do not write these up!
OTHER INTERESTING PROBLEMS: Section 2.2: # 5, 7, 8, 17.
Do not write these up!