Friday 9^{th} December, 2016 13:50

Final topics

New material

(1) Min-cost flow - algorithm cycle 7.2
(2) incremenatal weighted flow network $N'(f)$
(3) Floyd-Warshall Algorithm section 6.5, meaning of $d_{i,k}^j$, D^j and E^j matrices, reconstructing a short path using E^n matrix - see handout.

Material from Exams 1 through 4

(1) Simplex method
(2) Lexicographic simplex - know the row selection rule pivot rules and how to apply it - Theorem 14.1
(3) Bland's rule - know the column and row selection rules and how to apply them
(4) Two phase simplex method - make sure you know how to transition between the two phases and what the solution to the first phase LP implies about the original LP.
(5) Fact that lexicographic simplex and Bland's rule do not cycle
(6) Formulas for pre-multiplication matrix and tableau (see presentation on simplex).
(7) Theorem 1 - see handout
(8) Fundamental theorem:
Suppose (P) is an LP is in standard form and the constraint matrix has full rank.
   - If (P) has a feasible solution, then it has a basic feasible solution.
   - If (P) does not have an optimal solution, then the LP is infeasible or unbounded.
   - If (P) has an optimal solution, then it has a optimal basic feasible solution.

(9) Definitions (from exam 1):
   1) linear function
   2) affine function
   3) feasible solution
   4) feasible region,
   5) optimal solution/optimum
   6) infeasible
   7) unbounded.
   8) general form
   9) standard form
   10) canonical form
   11) slack variable
   12) surplus variable
   13) full rank
   14) basis
   15) feasible basis
   16) basic solution
   17) basic feasible solution,
   18) solved for a basis
   19) basic variables
   20) nonbasic variables
   21) tableau
   22) pre-multiplication matrix
   23) relative cost of a variable/column
   24) relative cost vector
   25) degenerate basic feasible solution
   26) zero-level
   27) cycling
   28) lexicographically positive/negative/zero
   29) lexicographically greater than/equal to/less than

30) artificial variables
31) driving artificial variables out of the basis

(10) Weak Duality - If $\mathbf{x}$ is feasible for P and π is feasible for its dual D, then $\pi^T \mathbf{b} \leq \mathbf{c}^T \mathbf{x}$ (and proof).
(11) Finding the dual of a linear program in general form (Definition 3.1)
(12) Strong Duality (Theorem 3.1)
(13) Dual of the dual is primal (Theorem 3.2)
(14) Chart with possible combinations of finite optimum, unbounded and infeasible for the primal and the dual (Theorem 3.3 in PS)
(15) Complementary slackness (section 3.4) (and proof)
(16) Farkas Lemma (Theorem 3.5) (and proof)
(17) Feasible basis and dual feasible basis - B is a dual feasible basis if $\pi^T = \mathbf{c}_B^T \mathbf{A}_B^{-1}$ is feasible for the dual
(18) Dual simplex method (Section 3.6)
(19) Handling additional constraints, changes to the objective function or changes to the vector $\mathbf{b}$.
(20) Shadow/marginal prices - (see the dual simplex presentation for a discussion of this topic - HW 6 #1 is also relevant).
(21) zero-sum games, pure strategy, mixed strategy, Alice, Bob, $\beta(\mathbf{x})$, $\alpha(\mathbf{y})$, worst-case optimal strategy, value of game, linear program corresponding to a matrix game, **Nash equilibrium**,
(22) Minimax theorem
(23) incidence matrix of a graph
(24) maximum matching as an ILP and minimum vertex cover as an ILP (see pages 146 and 147 in section 8.2 of GM)
(25) Integer programming and satisfiability
(26) Totally unimodular matrices (definition)
(27) Incidence matrix of graphs and directed graphs (definition)
(28) Theorem 13.3 (and proof)
(29) The incidence matrix of bipartite graphs and directed graphs are TUM (Theorem 13.3 Corollary)
(30) (Köning's theorem) Min vertex cover = max matching in bipartite graphs - via integer programming
(31) Relaxation of an integer linear program
(32) When the matrix associated with an LP is TUM, b is integer and the LP has a finite optimal solution, there exists an optimal solution that is integer. If c is integer, then the dual has an optimal solution that is integer
(33) If A is TUM and b is integer the basic feasible solutions of the LP in standard form are integer
(34) Branch and bound
(35) Network (definition)
(36) Max flow LP (first formulation - not section 4.3)
(37) shortest path and its dual (Section 3.4),
(38) circulation/flow/value of a flow (definitions) -
(39) A circulation is the sum of flows on cycles, and a (s,t)-flow is the sum of flows cycles and flows on (s,t)-paths
(40) revised simplex method and two phase revised simplex (section 4.1)
(41) Max-flow via revised simplex (Section 4.3)
(42) Primal dual algorithm, Theorem 5.1 - with proof, Theorem 5.3 - with proof, Theorem 6.1 - with proof
(43) Primal dual for shortest path (Section 5.4)
(44) Primal dual for max flow - Ford-Fulkerson algorithm
(45) Finding an f-augmenting (s,t)-path
(46) Definition of an (s,t)-cut and the capacity of an (s,t)-cut (Definition 6.1)
(47) Max flow LP and its Dual LP
(48) Feasible solution to dual of max flow LP corresponding to an (s,t)-cut (Theorem 6.1) (with proof)
(49) Finding a minimum (s,t)-cut in a network
(50) Max flow equals min cut theorem (Theorem 6.2)