Math 482 **HW10** Section: ________ Name: ____________________

Part One (Questions 1 and 2) Due Fri Dec 2, 2016
Part Two (Questions 3, 4 and 5) Due Wed Dec 7, 2016

Students in section E13 (three credit hours) need to solve any four of the following five problems. (You can do 1 problem form Part one and 3 problems form part two or 2 problems from part one and 2 problems from part two.) Students in section E14 (four credit hours) must solve all of the problems.

Part One - Due Fri Dec 2, 2016

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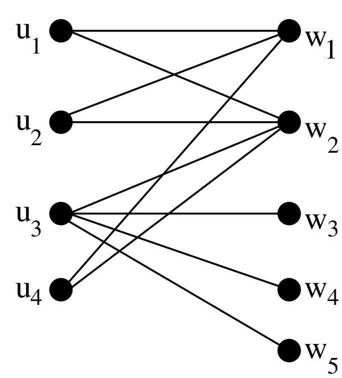
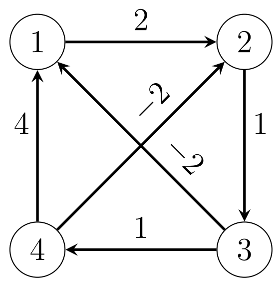
1. The president of a large university with $3k$ (k is an integer) academic departments and ℓ professors is appointing a committee. One professor will be chosen from each department. Many professors have joint appointments in two or more departments, but none can be designated representative of more than one department. The president also wants equal representation among assistant professors, associate professors and full professors. Design a flow problem to produce the desired committee if possible.

2. A matching is a set of edges such that no vertex is contained in more than one edge. Using maximum flows, find a maximum matching in the bipartite graph below on the left. Prove that the matching is optimal. *Hint: Construct an network in which a flow of value v corresponds to a matching in the given graph that contains v edges. Then find a maximum flow in this network and prove that it is a maximum flow by finding a minimum cut in the network. You should* ***clearly*** *write the network you are using, the maximum flow and the minimum cut in this network.*

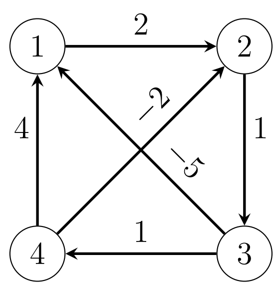


Part Two - Due Wed Dec 7, 2016

3. Find the length of a shortest paths between every pair of vertices in the following directed graph using the Floyd–Warshall algorithm. Your answer should include the matrices D^j and E^j for every j from 0 to 4. Then, write the steps necessary to reconstruct the shortest path from node 4 to node 1 using the matrix E^4.



4. Use the Floyd-Warshall algorithm to find a negative cost cycle in the following directed graph. Your answer should include all of the matrices D^j and E^j that you constructed and should describe how you used the matrices to detect and reconstruct a negative cost cycle.



5. Let N be the network consisting of the directed graph with edges $E = [sa, sb, ac, ba, bc, cb, ct, bt]$, capacities $b = [2, 4, 3, 3, 2, 2, 3, 3]^T$, costs $c^T = [2, 3, 1, 2, 2, 1, 1, 5]$ and let $f = [2, 3, 3, 1, 0, 0, 3, 2]^T$ be a flow. This network and flow is drawn below. Our aim is to construct a min-cost flow with value 5.

   (a) Compute the cost of the given flow.

   (b) Draw the incremental weighted flow network $N'(f)$ and find a negative cost f^r cycle in $N'(f)$ (you do not need to use the Floyd-Warshall algorithm for this step).

   (c) Then find the maximum θ such that $f^* = f + \theta f^r$ is a feasible flow.

   (d) Determine if the flow $f^* = f + \theta f^r$ is a minimum cost flow with value 5.