Math 482 **HW7** Section: __________ Name: ______________________________

Due Wednesday, October 26, 2016

Students in section D13 and E13 (three credit hours) need to solve any four of the following five problems. Students in section D14 and E14 (four credit hours) must solve all five problems.

1. Consider the following integer program P:

$$\begin{array}{rccrl} & z = x_1 & \to & & \min \\ \text{subject to} & 3x_1 & -100x_2 & \geq & 1 \\ & 3x_1 & -101x_2 & \leq & 1 \\ & x_1, & x_2 & \geq & 0 \\ & x_1, & x_2 & & \text{integer} \end{array}$$

   Solve the linear programming relaxation of P, obtaining an optimal solution x^* with cost z^* (You can solve the linear programming relaxation in any manner that you wish). Obtain an integer vector x from x^* by rounding each component to the nearest integer. Is x an optimal solution to the integer program P? If it is not, find an optimal solution to the integer program P.
   Hint: Since the objective function of the integer linear program is just x_1, you can find an optimal solution by checking if there is a feasible solution in which $x_1 = 0$, then checking if there is a feasible soluition in which $x_1 = 1$, etc.

2. Prove that if $\mathbf{A}$ is TUM, then

   1. $\mathbf{A}^T$ is TUM,
   2. $(\mathbf{A}|\mathbf{e}_i)$ it TUM, for all $i \in [m]$, and
   3. $(\mathbf{A}|\mathbf{A}_j)$ is TUM, for all $j \in [n]$.

3. Show that every matrix with entries $-1, 0$ and 1 such that at most one row has more than one non-zero entry is totally unimodular. Give an example of a totally unimodular matrix with at least three non-zero elements in every column and in every row.

4. Solve the INTEGER linear program below using the branch and bound algorithm.

$$\text{Maximize } z = x_1 + x_2$$

   subject to

$$\begin{cases} 4x_1 + x_2 & \leq & 16, \\ -2x_1 + 5x_2 & \leq & 10, \\ x_1, x_2 & \geq & 0, \\ x_1 \in \mathbb{Z}, & x_2 \in \mathbb{Z} & \end{cases}$$

   Draw the feasible regions of all feasible nodes in the plane $0x_1x_2$.

5. Consider the following integer program:

$$\begin{array}{llll} \text{Maximize} & 10x_1 & +8x_2 & \\ \text{subject to} & 11x_1 & +7x_2 & \leq 38 \\ & 7x_1 & +9x_2 & \leq 35 \\ & x_1, & x_2 & \geq 0 \\ & x_1, & x_2 & \in \mathbb{Z}, \end{array}$$

and let P be its integer relaxation. Suppose that while solving this program with the branch and bound method you encounter the following nodes:

Node name	Added constraint	Solution	
$P:$		$x^* = (97/50, 119/50)^T$,	$z^* = 1922/50 \approx 38.4$
$PA:$	$x_1 \geq 2$,	$x^* = (2, 16/7)^T$,	$z^* = 268/7 \approx 38.3$
$PAA:$	$x_2 \geq 3$,	infeasible	
$PAB:$	$x_2 \leq 2$,	$x^* = (24/11, 2)^T$,	$z^* = 416/11 \approx 37.8$
$PABA:$	$x_1 \geq 3$,	$x^* = (3, 5/7)^T$,	$z^* = 250/7 \approx 35.7$
$PABAA:$	$x_2 \geq 1$,	infeasible	
$PABAB:$	$x_2 \leq 0$,	$x^* = (38/11, 0)$,	$z^* = 380/11 \approx 34.5$
$PABABA:$	$x_1 \geq 4$,	infeasible	
$PABABB:$	$x_1 \leq 3$,	$x^* = (3, 0)$,	$z^* = 30$
$PABB:$	$x_1 \leq 2$,	$x^* = (2, 2)^T$,	$z^* = 36$
$PB:$	$x_1 \leq 1$,	$x^* = (1, 28/9)^T$,	$z^* = 314/9 \approx 34.9$
$PBA:$	$x_2 \geq 4$,	infeasible	
$PBB:$	$x_2 \leq 3$,	$x^* = (1, 3)^T$,	$z^* = 34$

What is the optimal solution of P? List the nodes that could kill other nodes and, for each such node, list the nodes that it kills. If a node kills another node that has descendants, you do not need to list the descendants only the parent node.