Math 482 **HW6** Section: ________ Name: ____________________

Due Friday, October 13, 2016

Students in section D13 and E13 (three credit hours) need to solve any four of the following five problems. Students in section D14 and E14 (four credit hours) must solve all five problems.

For questions 3 and 4, you can convert the LP to standard form and then solve using two-phase simplex, but I recommend that you use an LP solver. For example, I was able to use the solver at https://www.easycalculation.com/operations-research/simplex-method-calculator.php to solve both LPs. If you use this solver, note that all variables are automatically constrained to be non-negative, so you have to change the LP to account for this and that the objective function must contain all the variables even if the coefficient corresponding to the variable in the object function is 0. You do not need to write all of the simplex steps if you us a solver or if you do it by hand.

1. Suppose that you have the LP
$$(P) \qquad \min \quad \mathbf{c}^T\mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} = \mathbf{b},$$
a basis $B = (j_1, \ldots, j_m)$ such that
$$\mathbf{A}_B^{-1}\mathbf{b} = \begin{pmatrix} 18 & 12 & 8 & 9 & 31 \end{pmatrix}^T, \pi^T = \mathbf{c}_B^T\mathbf{A}_B^{-1} = \begin{pmatrix} 8 & 20 & 30 & 7 & 8 \end{pmatrix}, \text{ and } \pi^T\mathbf{b} = 159,$$
and that you know that the third column of $\mathbf{A}_B^{-1}$ is $\begin{pmatrix} 1 & 5 & -2 & 2 & 6 \end{pmatrix}^T$. Find the largest number Δ such that if $\mathbf{b}' = \mathbf{b} + \Delta\mathbf{e}_3$ ($\mathbf{e}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}^T$ is the third standard basis vector), then B is both a primal and dual feasible basis for the LP
$$(P') \qquad \min \quad \mathbf{c}^T\mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} = \mathbf{b}'.$$
Also, with the maximum value of Δ that was computed in the previous step, find the value of the objective function at an optimal solution of P'.

2. What is the worst-case optimal **pure** strategy for Alice and what is the expected payout assuming Bob plays optimally? In other words, find $i_0 \in [m]$ such that $\beta(\mathbf{e}_{i_0}) = \max_{i\in[m]} \beta(\mathbf{e}_i)$ where $\mathbf{e}_i$ is the ith standard basis vector and give the value of $\beta(\mathbf{e}_{i_0})$. Determine the corresponding information for Bob. (Hint: Use the fact that $\beta(\mathbf{x}) = \min_{j\in[n]} \mathbf{x}^T\mathbf{M}\mathbf{e}_j$ and $\alpha(\mathbf{y}) = \max_{i\in[m]} \mathbf{e}_i^T\mathbf{M}\mathbf{y}$.)
$$\begin{pmatrix} 4 & 8 & 1 & 2 & 6 & 7 \\ 8 & 2 & 3 & 5 & 9 & 2 \\ 1 & 8 & 1 & 7 & 3 & 3 \\ 6 & 5 & 6 & 8 & 6 & 3 \\ 2 & 6 & 2 & 4 & 7 & 1 \\ 3 & 9 & 1 & 5 & 2 & 6 \end{pmatrix}.$$

3. **Before attempting this problem please read the comments at the beginning of this homework.** Consider the following game. On each turn both Alice and Bob both hide either 1 or 2 dollars behind there backs and then, at they same time, they guess what the other player has hidden. If either both players guess the correct number or both players guess incorrectly, then no money changes hands. Otherwise, if one player guess correctly and the other player guess incorrectly, the incorrect player must pay the other player the **total** amount hidden by both players.

   More formally, Alice plays (h_1, g_1) and Bob plays (h_2, g_2) where $h_i, q_i \in \{1, 2\}$ for $i \in \{1, 2\}$. Note that each player has 4 pure strategies. If either both $g_1 = h_2$ and $g_2 = h_2$ or both $g_1 \neq h_2$ and $g_2 \neq h_1$, then the payout is 0. If $g_1 = h_2$ and $g_2 \neq h_1$, then the payout is $h_1 + h_2$ (Bob pays Alice $h_1 + h_2$ dollars), and if $g_1 \neq h_2$ and $g_2 = h_1$, then the payout is $-(h_1 + h_2)$ (Alice pays Bob $h_1 + h_2$ dollars).

   Write the matrix that corresponds to this game and find the value of the game, a worst-case optimal mixed strategy for Alice, and a worst-case optimal mixed strategy for Bob. You must write the LP you are solving **clearly** before giving the answer, and you **must** give the answer using fractions.

4. **Before attempting this problem please read the comments at the beginning of this homework.** Consider the following modification to the game presented in the previous question. Suppose that, after hiding their money, Bob always guesses first, so Alice always knows what Bob has guessed before she guesses. This changes the game as Alice has 4 additional pure strategies: she can either hide 1 or 2 dollars and then either repeat Bob's guess or guess differently than Bob. So Alice has 8 pure strategies in this game, while Bob still only has 4 pure strategies. Write the matrix that corresponds to this game and find the value of the game, a worst case optimal mixed strategy for Alice and a worst case optimal mixed strategy for Bob. You must write the LP you are solving **clearly** before giving the answer and you **must** give the answer using fractions.

5. Let $G = (V = \{v_1, \ldots, v_n\}, E = \{e_1, \ldots, e_m\})$ be a graph in which every vertex is in at least one edge, i.e. for every $v_i \in V$ there exists some $e_j \in E$ such that $v_i \in e_j$. A set $C \subseteq E$ is an edge cover if for every vertex $v_i \in V$, there exists some $e_j \in C$ such that $v_i \in e_j$. Formulate the problem of finding a minimum size edge cover as an integer linear program.