Math 482 **HW5** Section: __________ Name: ______________________________

Due Friday, October 7, 2016

Students in sections D13 and E13 (three credit hours) need to solve any four of the following five problems. Students in sections D14 and E14 (four credit hours) must solve all five problems.

1. Let (P) be the following LP

$$\min \qquad \mathbf{c}^T\mathbf{x} \text{ s.t } \mathbf{A}\mathbf{x} \geq -\mathbf{c} \text{ and } \mathbf{x} \geq \mathbf{0} \tag{P}$$

   where A is *skew-symmetric*, which means that $\mathbf{A} = -\mathbf{A}^T$. Prove that if there exist a feasible solution of P, then there exists an optimal solution $\mathbf{x}^*$ of P and $\mathbf{c}^T\mathbf{x}^* = 0$.

2. Prove the theorem due to P. Gordan (1873) that the system $\mathbf{Ax} < \mathbf{0}$ is unsolvable if and only if the system $\mathbf{y^T A} = \mathbf{0}, \quad \mathbf{y} \geq \mathbf{0}, \quad \mathbf{y} \neq \mathbf{0}$ is solvable. (Hint: In order to apply the duality theorems, replace the system $\mathbf{Ax} < \mathbf{0}$ of strict inequalities by the system $\mathbf{Ax} \leq -\mathbf{1}$ of non-strict inequalities. Prove that the new system is solvable if and only if $\mathbf{Ax} < \mathbf{0}$ is solvable).

3. **Note:** In an earlier version, the second constraint was $-x_1 + x_2 + x_3 + x_4 \geq 10$ and it has now been changed to $-x_1 + x_2 + x_3 - x_4 \geq 10$. In this version the computations should be simplier, but if you did the problem with the original constraints you do not need to redo the problems, just turn in the version you have completed. You should use the same version of the constraints for problems 3, 4 and 5.

   Use the dual simplex method to find an optimal solution to the problem

$$\text{Minimize } z = 7x_1 + x_2 + 3x_3 + x_4$$

   subject to

$$\begin{cases} 2x_1 - 3x_2 - x_3 + 2x_4 & \geq & 8, \\ -x_1 + x_2 + x_3 - x_4 & \geq & 10, \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{cases}$$

4. The additional constraint

$$6x_1 + x_2 + 2x_3 - 2x_4 \quad \geq 24,$$

   is added to those of Problem 3. Solve the new problem starting from the optimal tableau for Problem 3.

5. Suppose the objective function in problem 3 is changed to

$$\text{Minimize } z = x_2 + 3x_3 + x_4$$

   Solve the new problem starting from the optimal tableau for Problem 3.