Math 482 **HW4** Name: __________________________

Due Friday, September 30, 2016

Students in sections D13 and E13 (three credit hours) need to solve any four of the following five problems. Students in sections D14 and E14 (four credit hours) must solve all five problems.

1. State the dual to the following problem:

$$z = 3x_1 - x_2 - 2x_3 \quad \longrightarrow \quad \min$$

with respect to

$$\begin{cases} 6x_1 & -2x_2 & +3x_3 & & +2x_5 & \geq & -7, \\ -2x_1 & +3x_2 & & -2x_4 & & \leq & 6, \\ 2x_1 & +x_2 & -4x_3 & +x_4 & & = & 6, \\ x_1, & x_2, & & x_4 & & \geq & 0 \end{cases}$$

2. The Transportation Problem (known also as the Hitchcock problem) is as follows. There are m sources of some commodity, each with a supply of a_i units, $i = 1, \ldots, m$ and n terminals, each of which has a demand of b_j units, $j = 1, \ldots, n$. The cost of sending a unit from source i to terminal j is c_{ij} and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. We want to find a cheapest way to satisfy all demands. State this problem as an LP, and state the dual to this problem. *(Hint: Create a small example problem with actual numbers and then take the dual of the example problem. The general answer can be expressed in a compact form.)*

3. Use the complementary slackness condition to check whether the vector $[3, -1, 0, 2]^T$ is an optimal solution to the problem

$$\text{Maximize } z = 6x_1 + x_2 - x_3 - x_4$$

subject to

$$\begin{cases} x_1 & +2x_2 & +x_3 & +x_4 & \leq & 5, \\ 3x_1 & +x_2 & -x_3 & & \leq & 8, \\ & x_2 & +x_3 & +x_4 & = & 1, \\ & & x_3, & x_4 & \geq & 0. \end{cases}$$

4. Prove that if P is an LP in standard form, P has an optimal solution, and P has no degenerate optimal solutions, then there is a unique optimal solution to the dual of P. (Hint: Use the complementary slackness condition and the fact that if an LP in standard form has an optimal solution, then it has an optimal basic feasible solution)

5. Let P be the following linear program

$$\min \mathbf{c}^T\mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} = \mathbf{b} \text{ and } \mathbf{x} \geq \mathbf{0}$$

and suppose that P has an optimal solution. Consider the problem P' formed by changing $\mathbf{b}$ to some other vector $\mathbf{b}'$. Prove if P' has a feasible solution, then P' has an optimal solution.