Math 482 **HW2** Name: ______________________________

Due Friday, September 9, 2016

Students in sections D13 and E13 (three credit hours) should solve any four of the following five problems. Students in sections D14 and E14 (four credit hours) must solve all five problems.

1. A farmer has 100 acres of land which he has decided to make part arable and part grass. Part of the land may also lie fallow. He can make an annual profit of \$150/acre from arable land and \$100/acre from grass. Each year arable land requires 25 hours work per acre, and grass requires 10 hours work per acre. The farmer does not want to work more than 2000 hours in any year. How should he divide his land between arable and grass so as to maximize his annual profit? You should formulate the problem as a linear program and write down this formulation as part of your solution (You can solve the LP using any method that you want).

2. Solve the following LP using the simplex method. You should find an optimal feasible solution and the value of the objective function at the solution. Draw in the plane $\mathbf{0x_1x_2}$ the feasible region for the problem. In this picture, mark all of the points corresponding to the basic solutions arising at the steps of the simplex method. (You must use the simplex method to receive points for this problem).

$$\text{Maximize } z = 5x_1 + 3x_2$$

$$\text{subject to} \quad \begin{cases} x_1 & +x_2 & \leq & 5, \\ x_1 & -x_2 & \leq & 3, \\ & x_1, x_2 & \geq & 0. \end{cases}$$

3. Solve the LP represented by the tableau below using the simplex method. After you are finished, find the matrix $\mathbf{X}$ that you would have to premultiply the **given** tableau by in order to get the final tableau that you reached using the simplex method.

		x_1	x_2	x_3	x_4	x_5
$-z$	-6	2	0	-1	0	3
x_1	3	1	0	-1	1	4
x_2	2	0	1	0	1/2	2

Be careful. Note that $a_{0,1}$ (the circled entry) is not equal to 0.

4. Suppose that you are given the LP represented by the following tableau to solve.

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
$-z$	0	0	0	0	0	0	0	-5	-8	-5	-6	-7
x_1	4	1	0	0	0	0	0	1	1	1	1	1
x_2	6	0	1	0	0	0	0	2	2	-1	-3	5
x_3	4	0	0	1	0	0	0	2	2	3	0	0
x_4	6	0	0	0	1	0	0	-3	0	4	5	6
x_5	9	0	0	0	0	1	0	-9	3	-3	0	-1
x_6	4	0	0	0	0	0	1	-4	0	-2	-1	5

Suppose further that you given the following tableau that is only partially complete, the entries $a_{1,0}$ through $a_{6,0}$ are missing and the basis heading is also not listed. You can assume that the tableau below was produced using the simplex method.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
24	1	0	$\frac{7}{2}$	1	0	0	0	0	$\frac{21}{2}$	0	0
$*$	$\frac{5}{3}$	0	$-\frac{5}{6}$	$-\frac{1}{3}$	0	0	1	0	$-\frac{13}{6}$	0	$-\frac{1}{3}$
$*$	3	1	$-\frac{5}{2}$	0	0	0	0	0	$-\frac{11}{2}$	0	8
$*$	$-\frac{5}{3}$	0	$\frac{4}{3}$	$\frac{1}{3}$	0	0	0	1	$\frac{11}{3}$	0	$\frac{1}{3}$
$*$	1	0	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	1	1
$*$	20	0	$-\frac{23}{2}$	-4	1	0	0	0	$-\frac{67}{2}$	0	-5
$*$	$\frac{23}{3}$	0	$-\frac{23}{6}$	$-\frac{4}{3}$	0	1	0	0	$-\frac{67}{6}$	0	$\frac{14}{3}$

Use the given partially complete tableau to find an optimal basic feasible solution, the corresponding basis and the value of the objective function at the optimum **without doing any simplex pivots or computing the inverse of a 6x6 matrix. The amount of computation necessary to solve this problem is not excessive**.

5. Prove that if variable x_s is moved out of the basis of a linear program at some step of the simplex method, then on the **next** step x_s will not be moved back into the basis. (Note that a single variable can be moved in and out of the basis multiply times during the execution of the simplex method. This question only implies that a variable cannot leave the basis on a step and then enter the basis on the very next pivot step.)