Math 482 HW1

Due Friday, September 2, 2016

Students in sections D13 and E13 (three credit hours) should solve any four of the following five problems. Students in sections D14 and E14 (four credit hours) must solve all five problems.

1. A company produces three types of chemicals: chemical A, chemical B and chemical C. They sell chemical A for \$30 per barrel, chemical B for \$20 per barrel, and chemical C for \$10 per barrel. Chemical A requires .2 units of energy and 5 units of raw material to produce, Chemical B requires .3 units of energy and 6 units of raw material to produce and Chemical C requires .2 units of energy and 3 units of raw material to produce. Assume that all of the chemicals the company produces are sold. The company must produce at least 55 barrels of chemicals per day. It can use at most 7 units of energy per day and at most 120 units of raw material each day. The company wishes to maximize its profits. Formulate the appropriate linear program in standard form.

2. Solve the following problem, i.e. if the following program has a optimal solution you must find an optimal solution and compute the value of the objective function at the optimal solution, otherwise you must state whether the program is infeasible or unbounded. You should draw the feasible region in the plane.

$$z = 3x_1 + 2x_2 \quad \longrightarrow \quad \max$$

with respect to

$$\begin{cases} x_1 & +4x_2 & \leq & 12, \\ x_1 & +x_2 & \leq & 4, \\ 5x_1 & +2x_2 & \leq & 15, \\ & x_1, x_2 & \geq & 0. \end{cases}$$

3. State (but do not solve) the following LP in standard form. Note that the variable x_3 is not constrained to be nonnegative.

$$z = -x_1 + 2x_2 - 3x_3 \quad \longrightarrow \quad \max$$

with respect to

$$\begin{cases} 4x_1 & +2x_2 & +2x_3 & \leq & 3, \\ x_1 & +x_2 & +4x_3 & \geq & -7, \\ 2x_1 & -3x_2 & & \leq & 5, \\ x_1, & x_2, & & \geq & 0. \end{cases}$$

4. Suppose that there exists $\mathbf{x}_0 \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^n$ such that $\mathbf{x}_0$ is feasible for the following linear program P

$$\begin{array}{rl} \min & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

and $\mathbf{y}$ satisfies

$$\begin{aligned} \mathbf{c}^T\mathbf{y} &< 0 \\ \mathbf{A}\mathbf{y} &= \mathbf{0} \\ \mathbf{y} &\geq \mathbf{0}. \end{aligned}$$

Prove that P is unbounded.

5. Find $\mathbf{A}$, $\mathbf{b}$ and $c_1 \ldots, c_n$ such that both the linear program

$$\text{Minimize } z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to $\left\{ \begin{array}{rcl} \mathbf{A}\mathbf{x} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0} \end{array} \right.$

and the linear program

$$\text{Minimize } \xi = -c_1x_1 - c_2x_2 - \ldots - c_nx_n$$

subject to $\left\{ \begin{array}{rcl} \mathbf{A}\mathbf{x} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0}. \end{array} \right.$

have feasible solutions with arbitrarily small cost. In other words, given your choices for $\mathbf{A}$, $\mathbf{b}$ and $c_1, \ldots, c_n$, for an arbitrary $M \in \mathbb{R}$ you should be able to find two different vectors $\mathbf{x}, \mathbf{x}' \geq 0$ such that $\mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{x}' = \mathbf{b}$, and both $c_1x_1 + c_2x_2 + \ldots + c_nx_n < M$ and $-c_1x_1' - c_2x_2' - \ldots - c_nx_n' < M$.