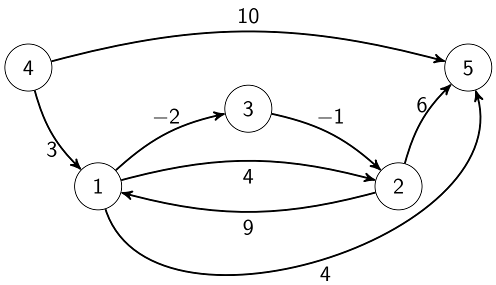
Directed graph with edge weights



Pop Quiz!

What are the following? $d^0_{4,3} = \infty$, $d^0_{4,5} = 10$, $d^0_{4,1} = 3$, $d^0_{1,5} = 4$, $d^1_{4,5} = 7$, $d^1_{4,2} = 7$, $d^1_{2,5} = 6$, $d^2_{4,5} = 7$, $d^2_{4,3} = 1$, $d^2_{3,5} = 5$, $d^3_{4,5} = 6$, $d^2_{1,1} = 13$, $d^2_{1,3} = -2$, $d^2_{3,1} = 8$, $d^3_{1,1} = 6$.

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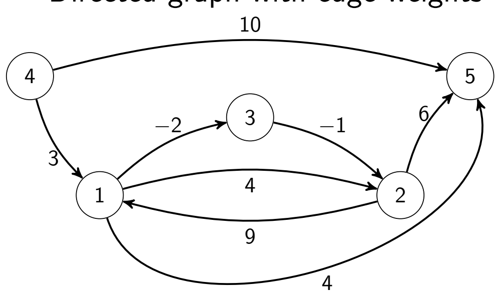


$$D^0 = \begin{pmatrix} \infty & 4 & -2 & \infty & 4 \\ 9 & \infty & \infty & \infty & 6 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & 10 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^0 = \begin{pmatrix} \infty & 4 & -2 & \infty & 4 \\ 9 & \infty & \infty & \infty & 6 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & 10 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^1 = \begin{pmatrix} \infty & 4 & -2 & \infty & 4 \\ 9 & 13 & 7 & \infty & 6 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^1 = \left(\begin{array}{c|c|ccc} \infty & 4 & -2 & \infty & 4 \\ \hline 9 & 13 & 7 & \infty & 6 \\ \hline \infty & -1 & \infty & \infty & \infty \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^2 = \begin{pmatrix} 13 & 4 & -2 & \infty & 4 \\ 9 & 13 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^2 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^2 = \left(\begin{array}{cc|c|cc} 13 & 4 & -2 & \infty & 4 \\ 9 & 13 & 7 & \infty & 6 \\ \hline 8 & -1 & 6 & \infty & 5 \\ \hline 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^2 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^3 = \begin{pmatrix} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 0 & 1 & \infty & 6 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^3 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^3 = \left(\begin{array}{ccc|c|c} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ \hline 3 & 0 & 1 & \infty & 6 \\ \hline \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^3 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^4 = \begin{pmatrix} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 0 & 1 & \infty & 6 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^4 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^4 = \left(\begin{array}{cccc|c} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 0 & 1 & \infty & 6 \\ \hline \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^4 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

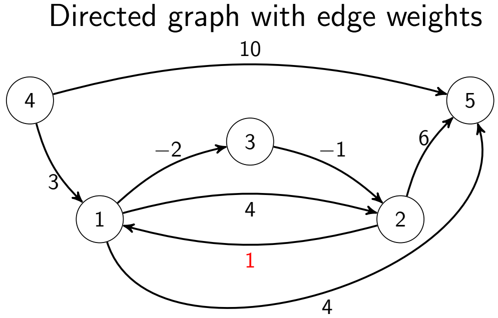
$$D^5 = \begin{pmatrix} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 0 & 1 & \infty & 6 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^5 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^5 = \begin{pmatrix} 6 & -3 & -2 & \infty & 3 \\ 9 & 6 & 7 & \infty & 6 \\ 8 & -1 & 6 & \infty & 5 \\ 3 & 0 & 1 & \infty & 6 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^5 = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

To reconstruct a shortest $(4,5)$-path

- Because $e^5_{45} = 3$, the vertex 3 is on a shortest $(4,5)$-path.
- Because $e^5_{43} = 1$, we know that 1 must appear appear after 4 and before 3 on this shortest path. Similarly, because $e^5_{35} = 2$, 2 must appear after 3 and before 5 on this path.
- Continuing, because $e^5_{41} = 0$, we know that no vertices appear between 4 and 1 on the shortest $(4,5)$-path. Similarly, because e^5_{13}, e^5_{32}, e^5_{25} are each 0, the path is exactly $4 \to 1 \to 3 \to 2 \to 5$ which has cost $d^5_{45} = 6$.

Directed graph with edge weights



$$D^0 = \begin{pmatrix} \infty & 4 & -2 & \infty & 4 \\ 1 & \infty & \infty & \infty & 6 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & 10 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^0 = \left(\begin{array}{c|cccc} \infty & 4 & -2 & \infty & 4 \\ \hline 1 & \infty & \infty & \infty & 6 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & 10 \\ \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^1 = \begin{pmatrix} \infty & 4 & -2 & \infty & 4 \\ 1 & 5 & -1 & \infty & 5 \\ \infty & -1 & \infty & \infty & \infty \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^1 = \left(\begin{array}{c|c|ccc} \infty & 4 & -2 & \infty & 4 \\ \hline 1 & 5 & -1 & \infty & 5 \\ \hline \infty & -1 & \infty & \infty & \infty \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{array}\right) \quad E^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D^2 = \begin{pmatrix} 5 & 4 & -2 & \infty & 4 \\ 1 & 5 & -1 & \infty & 5 \\ 0 & -1 & -2 & \infty & 4 \\ 3 & 7 & 1 & \infty & 7 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix} \quad E^2 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Because the **first** negative entry on a diagonal is $D^2_{3,3}$, there is a negative cost cycle that contains both the vertex 2 and the vertex 3. We can use E^1 to find the other vertices on this negative cost cycle. In this case, because $e^1_{32} = 0$, $e^1_{23} = 1$, $e^1_{21} = 0$ and $e^1_{13} = 0$, the negative cost cycle is $3 \to 2 \to 1 \to 3$.