Example (2.7 from book)

- ► We will use the following *pivot rules*. *Pivot rules* are additional rules for selecting the pivot row and the pivot column.
- Select the pivot column s so that $a_{0,s} = \overline{c}_s \leq \overline{c}_j = a_{0,j}$ for all $j \in [n]$. (In this example, this always gives a unique choice.) This was Dantzig's original rule for selecting a pivot column and is sometime called *Dantzig's rule* or the *largest coefficient rule*.
- ► In the case of ties when selecting the pivot row, select the row so that the smallest index leaves the basis. In other words, assuming the tableau is solved for the basis B = (j₁,..., j_m), select the pivot row r, from all valid choices, so that j_r is as small as possible. This always gives a unique choice.



bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis (5, 6, 7).

- 1. The bfs is degenerate.
- 2. x_5 and x_6 are basic variables at zero-level
- 3. We will bring x_1 into the basis (replacing x_5) at zero-level.

Tableau 2

			1							
			<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	<i>x</i> 6	<i>x</i> ₇	
	-z	3	$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0	
$T_1 =$	<i>x</i> 5	0	$\frac{1}{4}$	-8	-1	9	1	0	0	
	x ₆	0	$\frac{1}{2}$	-12	$-\frac{1}{2}$	3	0	1	0	
	<i>x</i> 7	1	0	0	1	0	0	0	1	
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$.										
						N.	¥~	¥-	16	
			<i>x</i> ₁	<i>x</i> ₂	X3	<i>X</i> 4	<i>X</i> 5	x ₆	<i>X</i> 7	
	- <i>z</i>	3	x ₁ 0	x ₂ -4	$\frac{X_3}{-\frac{7}{2}}$	x ₄ 33	<i>x</i> 5 3	<i>x</i> ₆	x ₇	
$T_2 =$	- <i>z</i> <i>x</i> ₁	3 0	x ₁ 0 1	x ₂ -4 -32	$\frac{x_3}{-\frac{7}{2}}$ -4	x ₄ 33 36	x ₅ 3 4	× ₆ 0 0	x ₇ 0 0	
$T_2 =$	-z x ₁ x ₆	3 0 0	x ₁ 0 1 0		$\begin{array}{r} x_3 \\ -\frac{7}{2} \\ -4 \\ \frac{3}{2} \end{array}$	x ₄ 33 36 -15	x ₅ 3 4 -2	x ₆ 0 0 1	x7 0 0 0	
$T_2 =$	-z x ₁ x ₆ x ₇	3 0 0 1	x ₁ 0 1 0 0	x_2 -4 -32 4 0	$ \begin{array}{r} x_3 \\ -\frac{7}{2} \\ -4 \\ \frac{3}{2} \\ 1 \end{array} $	$\frac{x_4}{33}$ 36 -15 0	$\frac{x_5}{3}$ 4 -2 0	x ₆ 0 1 0	x ₇ 0 0 0 1	

$T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{3} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_{5} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{3} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{1} & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{3} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{1} & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ x_{2} & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{2} & -\frac{1}{2} & \frac{1}{4} & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{1} & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{5} (0,0,0,0,0,0,1)^{T}, \text{ basis } (1,2,7).$ Tableau 4 $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{5} (0,0,0,0,0,0,1)^{T}, \text{ basis } (5,6,7).$ $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{3} & 0 & \frac{1}{6} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ x_{7} & 1 & -\frac{1}{6} & 0 & 0 & 2\frac{1}{2} & \frac{3}{2} & -1 & 1 \\ y_{7} & 1 & -\frac{1}{6} & 0 & 0 & 2\frac{1}{2} & \frac{3}{2} & -1 & 1 \\ y_{7} & 1 & -\frac{1}{6} & 0 & 0 & 2\frac{1}{2} & \frac{3}{2} & -1 & 1 \\ y_{7} & 1 & -\frac{1}{6} & 0 & 0 & 2\frac{1}{2} & \frac{3}{2} & -1 & 1 \\ y_{7} & 0 & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_{7} & 1 & -\frac{1}{6} & 0 & 0 & 2\frac{1}{2} & \frac{3}{2} & -1 & 1 \\ y_{7} & 1 & -\frac{1}{6} & 0 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 & 0 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_{7} & \frac{1}{1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ y_{7} & \frac{1}{1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ y_{7} & \frac{1}{1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ y_{7} & \frac{1}{1} & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ x_{7} & \frac{1}{1} & \frac{x_{2}}{6} & \frac{x_{3}} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{5} & -\frac{x_{3}}{2} & -\frac{1}{2} & \frac{16}{6} & 0 & 0 & -1 & 1 & 0 \\ x_{7} & \frac{1}{1} & \frac{x_{2}}{5} & -56 & 1 & 0 & 2 & -66 & 0 \\ x_{7} & 0 & -\frac{5}{5} & 56 & 1 & 0 & 2 & -66 $	Tableau 3										
$T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (5,6,7).$ $T_{3} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -x & 3 & 0 & 0 & -2 & 18 & 1 & 1 & 0 \\ \hline x_{1} & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ \hline x_{2} & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{7} & -\frac{1}{2} & \frac{1}{4} & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (1,2,7).$ $Tableau 4$ $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline \frac{x_{2}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (5,6,7).$ $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline \frac{x_{3}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (5,6,7).$ $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline \frac{x_{3}} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (3,2,7).$ $Tableau 5$ $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline \frac{x_{3}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0,0,0,0,0,0,1)^{T}, basis (5,6,7).$ $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{3} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{matrix}$				×.	¥-	¥-	¥.	¥-	¥-	¥-	
$T_{1} = \frac{\frac{1}{x_{0}} \left[\begin{array}{c} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}$		-7	3	$-\frac{3}{2}$	20	$-\frac{1}{2}$		0	0	0	
$T_{3} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{3}} = \frac{1}{3$	$T_1 =$	 X5	0	<u>4</u> <u>1</u>	-8	$\frac{2}{-1}$	9	1	0	0	
$\begin{aligned} \begin{array}{c} x_{7} \left 1 \right & \frac{2}{0} & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 \\ \text{bfs} (0,0,0,0,0,0,1)^{T}, \text{ basis} (5,6,7). \\ T_{3} &= \frac{\left \begin{array}{c c c c c c } x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & 0 & 0 & -2 & 18 & 1 & 1 & 0 \\ \hline x_{1} & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ \hline x_{2} & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline bfs (0,0,0,0,0,0,1)^{T}, \text{ basis} (1,2,7). \\ \end{array} \\ \hline \end{array} \\ \hline \begin{array}{c} \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \hline \\ \hline \end{array} \\ \hline \\ \hline$		х _б	0	$\frac{4}{1}$	-12	$-\frac{1}{2}$	3	0	1	0	
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$. $T_3 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline x_1 & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ x_2 & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{12} & -\frac{1}{2} & \frac{1}{4} & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(1, 2, 7)$. Tableau 4 $T_1 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$. $T_4 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(3, 2, 7)$. Tableau 5 $T_1 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_7 & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \hline bfs (0, 0, 0, 0, 0, 0, 1)^T$, basis $(3, 2, 7)$. Tableau 5		x ₇	1	0	0	1	0	0	0	1	
$T_{3} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{1} & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ x_{2} & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ -\frac{z} & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ -\frac{z} & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $F_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ -\frac{z} & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \end{bmatrix}$ $F_{5} (0,0,0,0,0,0,1)^{T}, \text{ basis } (3,2,7).$ $Tableau 5$ $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ x_{3} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ y_{7} & y_{7} &$	bfs (0,0,	0,0,	0,0	$(1)^{T}$, t	basis (5	,6,7).					
$T_{3} = \frac{\frac{-z}{x_{1}} \frac{3}{0} \frac{0}{0} \frac{-2}{18} \frac{18}{1} \frac{1}{10} \frac{1}{0} \frac{1}{0} \frac{0}{8} \frac{-284}{-12} \frac{1}{8} \frac{1}{0} \frac{1}{10} \frac{0}{0} \frac{1}{10} \frac{3}{8} \frac{-15}{-12} \frac{1}{2} \frac{1}{4} \frac{1}{0} \frac{1}{0$				<i>x</i> ₁	<i>x</i> 2	<i>x</i> 3	<i>X</i> 4	<i>X</i> 5	<i>x</i> 6	X7	
$T_{3} = \frac{T_{1}}{x_{2}} \begin{vmatrix} 1 & 0 & 1 & 0 & 8 & -84 & -12 & 8 & 0 \\ x_{2} & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (1, 2, 7).$ Tableau 4 $T_{1} = \frac{\frac{-z}{x_{3}} & \frac{3}{-\frac{3}{4}} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{-z}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{z_{7}}{x_{7}} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (5, 6, 7).$ $T_{4} = \frac{\frac{-z}{3} & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \frac{-z}{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \frac{x_{2}}{x_{3}} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \frac{x_{2}}{x_{7}} & 0 & \frac{1}{4} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (3, 2, 7).$ Tableau 5 $T_{1} = \frac{\frac{-z}{x_{3}} & \frac{x_{1}}{-\frac{3}{4}} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{x_{3}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{x_{6}}{x_{6}} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & 1 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{z_{7}}{x_{5}} & 0 & 1 & 0 & 0 & 0 & 1 \\ \frac{z_{7}}{x_{5}} & 0 & 1 & 0 & 0 & 0 & 1 \\ \frac{z_{7}}{x_{5}} & 0 & 1 & 0 & 0 & 0 & 1 \\ \frac{z_{7}}{x_{5}} & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ \frac{z_{7}}{x_{5}} & 0 & \frac{z_{7}}{x_{5}} & z$		- <i>z</i>	3	0	0	-2	18	1	1	0	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$T_3 =$	<i>x</i> ₁	0	1	0	8	-84	-12	8	0	
$\begin{aligned} x_{T} \mid 1 \mid 0 0 1 0 0 0 1 \\ bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (1, 2, 7). \end{aligned}$ Tableau 4 $T_{1} = \frac{1}{\frac{-z}{x_{5}}} \frac{x_{1}}{0} \frac{x_{1}}{2} \frac{x_{2}}{20} \frac{x_{3}}{1} \frac{x_{4}}{6} \frac{x_{5}}{0} \frac{x_{7}}{1} \frac{x_{7}}{0} \frac{x_{3}}{1} \frac{x_{2}}{20} \frac{x_{3}}{1} \frac{x_{4}}{0} \frac{x_{5}}{0} \frac{x_{7}}{1} \frac{x_{7}}{1} \frac{x_{7}}{0} \frac{x_{3}}{1} \frac{x_{1}}{0} \frac{x_{2}}{1} \frac{x_{2}}{3} \frac{x_{3}}{0} \frac{x_{4}}{1} \frac{x_{5}}{0} \frac{x_{7}}{1} \frac{x_{1}}{0} \frac{x_{2}}{0} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{5}}{0} \frac{x_{6}}{1} \frac{x_{7}}{1} \frac{x_{1}}{0} \frac{x_{2}}{0} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{2}}{0} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{5}}{2} \frac{x_{6}}{2} \frac{x_{7}}{1} \frac{x_{1}}{0} \frac{x_{2}}{1} \frac{x_{2}}{2} \frac{x_{3}}{2} \frac{x_{4}}{1} \frac{x_{5}}{1} \frac{x_{6}}{1} \frac{x_{7}}{1} \frac{x_{1}}{1} \frac{x_{2}}{1} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{5}}{2} \frac{x_{6}}{2} \frac{x_{7}}{1} \frac{x_{1}}{1} \frac{x_{2}}{1} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{5}}{2} \frac{x_{6}}{2} \frac{x_{7}}{1} \frac{x_{1}}{1} \frac{x_{2}}{1} \frac{x_{3}}{2} \frac{x_{4}}{2} \frac{x_{5}}{2} \frac{x_{6}}{2} \frac{x_{7}}{1} \frac{x_{1}}{1} \frac{x_{2}}{1} \frac{x_{3}}{1} \frac{x_{4}}{1} \frac{x_{5}}{1} \frac{x_{6}}{1} \frac{x_{7}}{1} \frac{x_{1}}{1} \frac{x_{2}}{1} x$		<i>x</i> ₂	0	0	1	<u>3</u> 8	$-\frac{15}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	0	
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(1, 2, 7)$. Tableau 4 $T_1 = \frac{1}{2} \begin{array}{c c c c c c } \hline x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline & & & & & & & & & & & & & & & & & &$		<i>X</i> 7	1	0	0	1	0	0	0	1	
Tableau 4 $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -x & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \end{bmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (3,2,7). Tableau 5 $T_{1} = \frac{\hline -\frac{-z}{x_{5}} & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{bmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\hline \frac{x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7}}{x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \\ \end{bmatrix}$	bfs (0,0,	0,0,	0,0	$(1)^{\mathcal{T}}$, k	basis (1	,2,7).					
$T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (5, 6, 7).$ $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{11}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (3, 2, 7).$ $Tableau 5$ $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (5, 6, 7).$ $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$	Tableau 4										
$T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (5, 6, 7).$ $T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (3, 2, 7).$ $Tableau 5$ $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ $bfs (0, 0, 0, 0, 0, 0, 1)^{T}, basis (5, 6, 7).$ $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$				V-	Va	Va	ν.	V-	Ve	¥-	
$T_{1} = \frac{\frac{1}{x_{5}} \begin{vmatrix} 3 & -\frac{4}{4} & -26 & -\frac{2}{2} & 6 & 6 & 6 & 6 & 0 \\ \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{vmatrix}$ $F_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{3}} & 0 & \frac{1}{6} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \end{vmatrix}$ $F_{5} (0, 0, 0, 0, 0, 0, 1)^{T}, \text{ basis } (3, 2, 7).$ $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{cases}$ $F_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{3}} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{bmatrix}$		_ 7	3	_ <u>3</u>	20	_ <u>1</u>	-74 6	^5 0	76 0	0	
$T_{1} = \frac{x_{5}}{x_{6}} \begin{bmatrix} 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ $fs (0,0,0,0,0,0,1)^{T}, \text{ basis } (5,6,7).$ $T_{4} = \frac{\frac{x_{1}}{x_{3}} \begin{bmatrix} x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{3}} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{bmatrix}$ $fs (0,0,0,0,0,0,1)^{T}, \text{ basis } (3,2,7).$ $Tableau 5$ $T_{1} = \frac{\frac{x_{1}}{x_{5}} \begin{bmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{5}} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ $fs (0,0,0,0,0,0,1)^{T}, \text{ basis } (5,6,7).$ $T_{5} = \frac{\frac{x_{1}}{x_{3}} \begin{bmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \frac{-z}{x_{3}} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \frac{x_{4}}{x_{7}} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & -2 & 6 & 1 \end{bmatrix}$	$T_1 =$		0	<u>4</u> 1	8	<u>2</u> _1	0	1	0	0	
$T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{4} & 0 & \frac{-z}{3} & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \hline bfs (0,0,0,0,0,0,1)^{T}, basis (3,2,7). \\ \hline Tableau 5$ $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline bfs (0,0,0,0,0,0,1)^{T}, basis (5,6,7). \\ \hline T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \\ \hline \end{array}$, 1	75 Ve	0	4 1	_0 _12	_1	3	1	1	0	
$T_{4} = \frac{x_{1} + x_{2} + x_{3} + x_{4} + x_{5} + x_{6} + x_{7}}{\frac{-z}{3} = \frac{1}{4} + \frac{x_{1} + x_{2} + x_{3} + x_{4} + x_{5} + x_{6} + x_{7}}{\frac{-z}{3} = \frac{1}{4} + \frac{x_{1} + x_{2} + x_{3} + x_{4} + x_{5} + x_{6} + x_{7}}{\frac{x_{2}}{3} + \frac{1}{6} +$		~6 ×7	1	2	0	- <u>2</u> 1	0	0	0	1	
$T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \\ \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (3,2,7). Tableau 5 $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{cases}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \\ \end{vmatrix}$	bfs (0,0	0,0,	0,0	1) ^T , t	basis (5	,6,7).	0	U	0	T	
$T_{4} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ \hline x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ \hline x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \hline x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{vmatrix}$ bfs $(0,0,0,0,0,0,1)^{T}$, basis $(3,2,7)$. $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs $(0,0,0,0,0,0,1)^{T}$, basis $(5,6,7)$. $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$,							
$T_{4} = \frac{\begin{vmatrix} -z & 3 & \frac{z}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\ x_{3} & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ x_{2} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ x_{7} & 1 & -\frac{1}{8} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (3,2,7). Tableau 5 $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$			2	1 x ₁	x ₂	X3	2 X4	x ₅	× ₆	X7	
$T_{4} = \frac{x_{3}}{x_{2}} \begin{bmatrix} 0 & \frac{1}{8} & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ \frac{x_{2}}{x_{7}} & 0 & -\frac{3}{64} & 1 & 0 & \frac{3}{16} & \frac{1}{16} & -\frac{1}{8} & 0 \\ \frac{1}{-\frac{1}{8}} & 0 & 0 & \frac{21}{2} & \frac{3}{2} & -1 & 1 \end{bmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (3,2,7). Tableau 5 $T_{1} = \frac{\frac{x_{1}}{x_{5}} & \frac{x_{1}}{-\frac{1}{4}} & \frac{x_{2}}{-\frac{3}{4}} & \frac{x_{4}}{20} & \frac{x_{5}}{-\frac{1}{2}} & \frac{1}{6} & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \frac{x_{6}}{x_{6}} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \frac{x_{6}}{x_{7}} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\frac{x_{1}}{x_{3}} & \frac{x_{1}}{-\frac{1}{2}} & \frac{x_{2}}{16} & \frac{x_{4}}{-\frac{1}{3}} & \frac{x_{5}}{2} & \frac{x_{6}}{-\frac{1}{2}} & \frac{x_{7}}{-\frac{1}{2}} & \frac{1}{2} & 0 & 0 \\ \frac{x_{4}}{x_{7}} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \frac{x_{7}}{x_{7}} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{bmatrix}$	т. —		3	<u>4</u> 1	0	1	-3 21	-2		0	
$\begin{aligned} x_2 & 0 & -\frac{1}{64} & 1 & 0 & 16 & 16 & -\frac{1}{8} & 0 \\ x_7 & 1 & -\frac{1}{8} & 0 & 0 & 21 & 2 & 3 & -1 & 1 \\ \end{bmatrix} \\ bfs (0,0,0,0,0,0,1)^T, basis (3,2,7). \end{aligned}$ $T_1 = \frac{ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_5 & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_6 & 0 & \frac{1}{2} & -12 & \frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{bmatrix} \\ bfs (0,0,0,0,0,0,1)^T, basis (5,6,7). \end{aligned}$ $T_5 = \frac{ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_3 & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_4 & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_7 & 1 & 5 & -56 & 0 & 0 & -2 & 6 & 1 \\ \end{aligned}$	14 —	<i>x</i> ₃	0	83	1	1	$-\frac{2}{3}$	$-\frac{1}{2}$	1	0	
$T_{7} 1 -\frac{1}{8} = 0 = 0 = \frac{1}{2} = \frac{1}{2} = -1 = 1$ bfs $(0,0,0,0,0,0,1)^{T}$, basis $(3,2,7)$. Tableau 5 $T_{1} = \frac{1}{\frac{-z}{x_{5}} = 0} = \frac{1}{\frac{x_{1}}{x_{2}} = \frac{x_{3}}{20} = \frac{x_{3}}{2} = \frac{x_{3}}{20} = \frac{1}{2} = 6}{\frac{1}{2} = 0} = 0 = 0$ bfs $(0,0,0,0,0,0,1)^{T}$, basis $(5,6,7)$. $T_{5} = \frac{1}{\frac{x_{1}}{x_{3}} = \frac{x_{2}}{2} = \frac{x_{3}}{20} = \frac{1}{2} = \frac{1}{$		<i>x</i> ₂	0	$-\frac{3}{64}$	1	0	$\frac{1}{16}$	$\frac{1}{16}$	- <u></u>	0	
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(3, 2, 7)$. Tableau 5 $T_1 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_5 & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_6 & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \end{vmatrix}$ bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$. $T_5 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_3 & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_4 & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_7 & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \\ \end{vmatrix}$		X7		-18	0	0	2	2	-1	1	
Tableau 5 $T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$	bfs (0, 0,	0,0,	0,0	$(1)^T$, k	basis (3	,2,7).					
$T_{1} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$	Tableau 5										
$T_{1} = \frac{\begin{vmatrix} -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_{5} & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ \hline x_{6} & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ \hline x_{7} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{vmatrix}$ bfs (0,0,0,0,0,0,1) ^T , basis (5,6,7). $T_{5} = \frac{\begin{vmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_{3} & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_{4} & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_{7} & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$				<i>x</i> 1	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	<i>x</i> 6	<i>X</i> 7	
$T_{1} = \frac{1}{\begin{array}{c cccccccccccccccccccccccccccccccccc$		- <i>z</i>	3	$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0	
$\begin{aligned} x_6 & 0 & \frac{1}{2} -12 -\frac{1}{2} 3 0 1 0 \\ x_7 & 1 & 0 0 1 0 0 0 1 \\ \end{bmatrix} \\ bfs \ (0,0,0,0,0,0,1)^T, \ basis \ (5,6,7). \\ T_5 &= \frac{ x_1 x_2 x_3 x_4 x_5 x_6 x_7}{ x_3 0 -\frac{5}{2} 56 1 0 2 -6 0 \\ x_4 & 0 -\frac{1}{4} \frac{16}{3} 0 1 \frac{1}{3} -\frac{2}{3} 0 \\ x_7 & 1 \frac{5}{2} -56 0 0 -2 6 1 \\ \end{bmatrix} \end{aligned}$	$T_1 =$	<i>X</i> 5	0	$\frac{1}{4}$	-8	-1	9	1	0	0	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		X ₆	0	1 2	-12	$-\frac{1}{2}$	3	0	1	0	
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$. $T_5 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_3 & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_4 & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_7 & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \\ \end{vmatrix}$		x ₇	1	0	0	1^2	0	0	0	1	
$T_5 = \frac{\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\ \hline x_3 & 0 & -\frac{5}{2} & 56 & 1 & 0 & 2 & -6 & 0 \\ \hline x_4 & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ \hline x_7 & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{vmatrix}$	bfs (0,0,	0,0,	0,0	1) ⁷ , t	basis (5	,6,7).					
$T_5 = \frac{\begin{array}{c cccccccccccccccccccccccccccccccccc$				<i>x</i> 1	<i>X</i> 2	X3	<i>X</i> 4	<i>X</i> 5	<i>×</i> 6	X7	
$T_{5} = \begin{array}{c ccccccccccccccccccccccccccccccccccc$		- <i>z</i>	3	$-\frac{1}{2}$	16	0	0	-1	1	0	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$T_5 =$	 X2	0	$-\frac{2}{5}$	56	1	0	2	-6	0	
$x_7 \mid 1 \mid \frac{5}{2} -56 0 0 -2 6 1$	-	Хл	0	$-\frac{1}{2}$	<u>16</u>	0	1	$\frac{1}{2}$	$-\frac{2}{2}$	0	
		ч Х7	1	4 5 2	3 56-	0	0	ئ 2-2	₃ 6	1	
		,		2	-						

Tableau 6									
			<i>X</i> 1	X2	X3	X4	<i>X</i> 5	×6	X7
	- <i>z</i>	3	$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0
$T_1 =$	X5	0	$\frac{1}{4}$	-8	-1	9	1	0	0
	x ₆	0	$\frac{1}{2}$	-12	$-\frac{1}{2}$	3	0	1	0
	<i>x</i> 7	1	0	0	1	0	0	0	1
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$.									
			<i>x</i> 1	<i>x</i> ₂	<i>x</i> 3	<i>X</i> 4	<i>x</i> 5	x ₆	<i>X</i> 7
	-z	3	$-\frac{7}{4}$	44	$\frac{1}{2}$	0	0	-2	0
$T_6 =$	<i>x</i> 5	0	$-\frac{5}{4}$	28	$\frac{1}{2}$	0	1	-3	0
	<i>x</i> ₄	0	$\frac{1}{6}$	-4	$-\frac{1}{6}$	1	0	$\frac{1}{3}$	0
	<i>X</i> 7	1	0	0	1	0	0	0	1
bfs (0,0,	0,0,	0,0	$,1)^{\mathcal{T}}$, k	basis (5	,4,7).				
Tableau 7	sam	ne a	as Tal	bleau	1!				
		1	I						
			<i>x</i> ₁	<i>x</i> ₂	X3	<i>X</i> 4	<i>x</i> 5	<i>x</i> ₆	<u> </u>
τ	-z	3	$-\frac{5}{4}$	20	- <u>+</u>	6	0	0	0
$I_1 =$	<i>x</i> 5	0	4 1	-8	-1	9	1	0	0
	<i>x</i> 6	0	2	-12	$-\frac{1}{2}$	3	0	1	0
	X7		0	0	T	0	0	0	T
bfs (0,0,	0,0,	0,0	,1) ⁷ , t	basis (5	, 6, 7).				
			<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>x</i> 5	x ₆	<i>x</i> 7
	- <i>z</i>	3	$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0
$T_7 =$	<i>X</i> 5	0	$\frac{1}{4}$	-8	-1	9	1	0	0
	x ₆	0	$\frac{1}{2}$	-12	$-\frac{1}{2}$	3	0	1	0
	<i>x</i> 7	1	0	0	1	0	0	0	1
bfs $(0, 0, 0, 0, 0, 0, 1)^T$, basis $(5, 6, 7)$.									
Example v	vith	le	xicog	raphic	simp	lex			
				24110	p				
	start	w:+	h tha a	ame ta	hlenu n		nla 0 7	from +	he book
but	this 1	time	e we fo	llowing	the lex	icograp	whic sin	nplex n	nethod.



Tableau 3

			<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>x</i> 5	x ₆	<i>x</i> 7
	-z	3	0	0	-2	18	1	1	0
$T_3 =$	x_1	0	1	0	8	-84	-12	8	0
	<i>x</i> ₂	0	0	1	<u>3</u> 8	$-\frac{15}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	0
	<i>X</i> 7	1	0	0	1	0	0	0	1

- ▶ We select 3 as the pivot column, it is the only possible pivot column
- ▶ In this step, rows 1 and 2 are tied, and we select row 2 because row 2 divided by $a_{2,3}$ is lexicographically less than row 1 divided by $a_{1,3}$.

Tableau 4 *x*₄ x_1 *x*7 x_2 X3 *X*5 *x*6 3 0 16/3 0 $^{-2}$ -5/37/3 0 7 $T_4 =$ -64/30 -4/30 1 -4 8/3 0 x_1 0 8/3 -4/30 1 - 10*x*3 2/30 1 0 -8/3 0 10 4/3 -2/31 X7 ▶ We pick column 5 as our pivot column. • Once we select column 5 as our pivot column, we must pivot on row 3 Tableau 5 X4 X5 *X*7 Х3 *x*6 2 0 21/2 0 3/2 5/4 17/40 $T_5 =$ 0 6 1 1 -240 2 1 x_1 0 0 0 1 0 0 1 *x*3 1 0 15/2 1 - 1/2 - 3/43/4 0 -2*X*5

Now we cannot select a pivot column.

► The bfs (1,0,1,0,3/4,0)^T with associated basis (1,3,5) is the optimal solution, because the entries in the top row (columns 1 through 7) are non-negative. In other words, the relative cost vector c^T is non-negative.

Quiz

	simplex										
		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	x ₆	<i>x</i> 7			
- <i>z</i>	8	2	0	0	2	-3	0	0			
X7	2	2	0	0	3	4	0	1			
x ₆	6	10	0	0	4	12	1	0			
x3	4	5	0	1	2	8	0	0			
x ₂	2	0	1	0	4	3	0	0			

► Find the pivot entry using Bland's rule and lexicographic

▶ Using Bland's rule we select column x₅ and row 3

• Using lexicographic simplex we select column x_5 and row 1