Homework 5
M588, 2015 SPRING
DUE: May 6 (W)

NAME:..
SCORE:..

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1. Find an element with maximum cardinality in the intersection of the two graphic matroids of Figure 12-16 of the textbook.

2. We are given a graph $G = (V, E)$ and a set $L \subseteq V$. We wish to determine whether there is a spanning tree of G such that nodes in L are all leaves of T (i.e. there can be leaves of T that are not in L, but every element of L must be a leaf of T). Formulate this problems as the intersection of a partition matroid and a graphic matroid. (You can assume that the number of vertices in G is is greater than 2).

3. Let $G = (V, E)$ be a bipartite graph. A set $S \subseteq V$ is a vertex cover if every edge in E has at least one end point in S. Use Edmond's matroid intersection theorem to prove that the maximum size of a matching in G is equal to the size of a minimum vertex cover. Recall that Edmond's matroid intersection theorem states that if $M_1(X, \mathcal{I}_1)$ and $M_2(X, \mathcal{I}_2)$ are two matroids then

$$\max_{Y \in \mathcal{I}_1 \cap \mathcal{I}_2} |Y| = \min_{U \subseteq X} (r_{M_1}(U) + r_{M_2}(X \setminus U))$$

4. Reformulate the dynamic programming algorithm for the TSP that was given in class (Example 18.8 in the text book) as a branch and bound algorithm with no upper or lower bounds, but with a dominance relation. For this problem, you need to describe the set of feasible solutions considered at each node, the set of children of each node and the dominance relation between nodes.

5. Solve the INTEGER linear program below using Gomory's cutting plane algorithm:

$$\text{Minimize } z = -4x_1 + x_2$$

subject to
$$\begin{cases} 7x_1 - 2x_2 & \leq & 14, \\ 2x_1 - 2x_2 & \leq & 3, \\ x_2 & \leq & 3, \\ x_1, x_2 & \geq & 0 \quad \text{and integer.} \end{cases}$$

Illustrate all cuts by pictures in the plane $0x_1x_2$. You do not need to show all intermediate calculations.