Homework 4
M588, 2015 SPRING
DUE: April 17 (F)

NAME:..
SCORE:...

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1. Given the following network with edges $[sa, sb, ac, ba, bc, cb, ct, bt]$, capacities $[2, 4, 3, 3, 1, 2, 4, 3]$, costs $[2, 3, 1, 2, 2, 1, 1, 5]$. The aim is to have a min-cost flow with value 5.
(i) Write this problems as a dual in standard form.
(ii) Given the following flow, construct the incremental weighted flow network N', and write the RP of this problem: The flowvector: $[2, 3, 3, 1, 1, 1, 3, 2]$.
(iii) Write down the DRP problem. Solve the DRP problem (you can just provide the solution, you do not have to show your work).
(iv) Given the solution for the DRP, improve the solution for the D, show the improved flow.

2. Use network flows to prove that a graph G is connected if and only if for every partition of $V(G)$ into two non-empty sets S, T, there is an edge with one endpoint in S and one endpoint in T.

3. The president of a large university with k academic departments is appointing a committee. One professor will be chosen from each department. Many professors have joint appointments in two or more departments, but none can be designated representative of more than one department. The president also wants equal representation among assistant professors, associate professors and full professors. (Assume that k is divisible by 3.). Design a flow problem to produce the desired committee.

4. Solve the following Hitchcock problem. Show your work:
Supplies: $a_1 = 3, a_2 = 2, a_3 = 1, a_4 = 4$. Demands: $b_1 = 1, b_2 = 3, b_3 = 2, b_4 = 2, b_5 = 2$. The cost-matrix is (listed row-wise): $[3, 2, 3, 1, 2]$, $[1, 5, 4, 5, 2]$, $[4, 4, 3, 2, 1]$, $[5, 1, 3, 5, 2]$.

5. In the alphabeta algorithm, prove that if in the flow problem associated with the RP, there exists a flow f that is optimal (i.e. there is no augmenting s,t-path in $N'(f)$), then the following is a solution to the dual of the restricted primal where

$$W := \{x \in V : \text{ there exists a path from } s \text{ to } x \text{ in the network } N'(f)\ \}$$

and $\overline{W} = V \setminus W$:

$$\begin{cases} \overline{\alpha}_i = 1 & \text{if source } i \in W \\ \overline{\alpha}_i = -1 & \text{if source } i \in \overline{W} \\ \overline{\beta}_j = -1 & \text{if terminal } j \in W \\ \overline{\beta}_j = 1 & \text{if terminal } j \in \overline{W} \end{cases}$$