Homework 3
M588, 2015 SPRING
DUE: April 1 (W)

NAME:..
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1. For $D \subseteq \mathbb{R}^n$ we define $co(D)$ to be the intersection of all convex sets containing D. Prove the following: Let $D \subseteq \mathbb{R}^n$. Then $co(D)$ coincides with the set

$$C := \{\lambda_1 x_1 + \cdots + \lambda_p x_p : x_1, \ldots, x_p \in D \text{ and } \lambda_1, \ldots, \lambda_p \geq 0 \text{ and } \lambda_1 + \cdots + \lambda_p = 1\}$$

Hint: Use the following steps.
1) Show that C is a convex set containing D.
2) Show that if B is a convex set containing D, then $C \subseteq B$.
3) Conclude that $co(D) = C$.

2. Consider the following 2-player zero-sum game: Each player hides either \$1 or \$2 and then, at the same time, they guess the amount of money the other player has hidden. If exactly one of the two players guesses correctly, then that player wins the *total* amount hidden. Otherwise, no money changes hands. For example, suppose Alice hides \$2 and Bob hides \$1. If Alices guesses that Bob has hidden \$1 and Bob guesses than Alice has hidden \$1 then Alice wins \$3 from Bob. If again Alice hides \$2 and Bob hides \$1 and Alice guesses that Bob has hidden \$1 and Bob guesses than Alice has hidden \$2 then no money changes hands. Write the matrix that corresponds to this game and find a worst case optimal mixed strategy for Alice using linear programming (You can/should use a computer to compute the solution). Suppose that, after hiding, Bob always guesses first, this changes the game as Alice has additional pure strategies (she can just repeat Bob's guess or guess differently than Bob). Find the matrix that corresponds to this modified game and find a worst case optimal mixed strategy for Alice. Also, compute the value of the game, i.e. the expected payout at the mixed Nash equilibrium.

3. Show that every matrix with entries $-1, 0$ and 1 such that at most one row has more than one non-zero entry is totally unimodular. Give an example of a totally unimodular matrix with at least three non-zero elements in every column and in every row.

4. From the solution of the max-flow problem by the simplex algorithm, prove he following: In any max-flow problem for a network with m arc, there is an optimal flow that can be decomposed into the sum of flows along no more than m chains.

5. Suppose there are n men and n women and m marriage brokers (labeled $c_1, \ldots, c_m$). Each broker has a list of men and women as clients and can arrange marriages between any pairs of men and women on the list. In addition, we restrict the number of marriages that broker i can arrange to a maximum of b_i. Each man can be married to at most one women and each women can be married to at most one man. Translate the problem of finding a solution with the most marriages into one of finding the maximum flow in a flow network.