HW1, M588, 2015 SPRING

DUE: February 16 (M) NAME:.............................

SCORE:.............

Please write each solution on a separate (new) page.

(1) Show by an example that 2-change does not define an exact neighborhood for the TSP.

(2) Does the fact that every basic feasible solution of an LP is nondegenerate imply that the solution is unique? If so prove it, otherwise give a counterexample.

(3) Solve the following linear program using the simplex method.

$$\text{Minimize } z = -5x_1 - 5x_2 - 3x_3$$

$$\text{subject to} \quad \begin{cases} x_1 & +3x_2 & +x_3 & \leq & 3 \\ -x_1 & & +3x_3 & \leq & 4 \\ 2x_1 & -2x_2 & +2x_3 & \leq & 4 \\ 2x_1 & +3x_2 & -x_3 & \leq & 2 \\ & & x_1, x_2, x_3 & \geq & 0. \end{cases}$$

(4) Show by an example that there can exist a degenerate basic feasible solution whose corresponding basis is unique.

(5) Prove that if variable x_s is moved out of the basis of a linear program at some step of the simplex method, then at the next step it will NOT be moved back into the basis.

(6) We are given the following two standard LPs (we assume only that A is an $m \times n$ matrix with rank m, where $m < n$. Is it possible that both LPs have arbitrary small feasible solution at the same time?

$$\min c'x, \quad Ax = b, \quad x \geq 0.$$

$$\min -c'x, \quad Ax = b, \quad x \geq 0.$$