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- It is not obvious how to find one or that a feasible solution even exists.

Suppose we are given the linear program

- Find a basic feasible solution?
- ► (0,0,0,4,-5,-1)^T is not a bfs!!!!! It has negative entries.
- It is not obvious how to find one or that a feasible solution even exists.
- We need the two-phase simplex method!

First step in the two phase simplex method: Make sure $b \ge 0$. How can we do this?

$$\begin{cases}
\text{Maximize } z = x_1 - x_2 + x_3 \\
2x_1 - x_2 + 2x_3 + x_4 &= 4 \\
2x_1 - 3x_2 + x_3 + x_5 &= -5 \\
-x_1 + x_2 - 2x_3 + x_6 &= -1 \\
x_1, x_2, x_3, x_4, x_5 x_6 \geq 0.
\end{cases}$$

First step in the two phase simplex method: Make sure $b \ge 0$. How can we do this? Multiply row *i* by -1 if $b_i < 0$. Note: We do not change the constraints by doing this.

 $\begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4\\ 2x_1 & -3x_2 & +x_3 & +x_5 & = & -5\\ -x_1 & +x_2 & -2x_3 & & +x_6 & = & -1\\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \ge & 0. \end{cases}$

First step in the two phase simplex method: Make sure $b \ge 0$. How can we do this? Multiply row *i* by -1 if $b_i < 0$. Note: We do not change the constraints by doing this.

 $\begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4\\ -2x_1 & +3x_2 & -x_3 & -x_5 & = & 5\\ x_1 & -x_2 & +2x_3 & & -x_6 & = & 1\\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \ge & 0. \end{cases}$

$$\begin{cases}
\text{Maximize } z = x_1 - x_2 + x_3 \\
\begin{cases}
2x_1 - x_2 + 2x_3 + x_4 &= 4 \\
-2x_1 + 3x_2 - x_3 &- x_5 &= 5 \\
x_1 - x_2 + 2x_3 &- x_6 &= 1 \\
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\end{cases}$$

1. is already solved for a basis, and

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- 1. is already solved for a basis, and
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$$\begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4 \\ -2x_1 & +3x_2 & -x_3 & -x_5 & = & 5 \\ x_1 & -x_2 & +2x_3 & & -x_6 & = & 1 \\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \ge & 0. \end{cases}$$

- 1. is already solved for a basis, and
- 2. whose optimal solution will either
- 3. give us a basic feasible solution for the original LP, or

$$\begin{array}{rcl} \text{Maximize } z = x_1 - x_2 + x_3 \\ \begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4 \\ -2x_1 & +3x_2 & -x_3 & -x_5 & = & 5 \\ x_1 & -x_2 & +2x_3 & & -x_6 & = & 1 \\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \ge & 0. \end{cases}$$

- 1. is already solved for a basis, and
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- 3. give us a basic feasible solution for the original LP, or
- 4. prove that the original LP was infeasible.

$$\begin{array}{rcl} \text{Maximize } z = x_1 - x_2 + x_3 \\ \begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4 \\ -2x_1 & +3x_2 & -x_3 & -x_5 & = & 5 \\ x_1 & -x_2 & +2x_3 & & -x_6 & = & 1 \\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \geq & 0. \end{cases}$$

- 2. whose optimal solution will either
- 3. give us a basic feasible solution for the original LP, or
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Add y_1 to equation 2 and y_2 to equation 3.

$$\begin{array}{rcl} & \text{Maximize } z = x_1 - x_2 + x_3 \\ \begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = & 4 \\ -2x_1 & +3x_2 & -x_3 & -x_5 & = & 5 \\ x_1 & -x_2 & +2x_3 & & -x_6 & = & 1 \\ x_1, & x_2, & x_3, & x_4, & x_5 & x_6 & \geq & 0. \end{cases}$$

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Maximize
$$\xi = -y_1 - y_2$$

$$\begin{cases} 2x_1 & -x_2 & +2x_3 & +x_4 & = 4 \\ -2x_1 & +3x_2 & -x_3 & -x_5 & +y_1 & = 5 \\ x_1 & -x_2 & +2x_3 & -x_6 & +y_2 & = 1 \\ x_1, & x_2, & x_3, & x_4, & x_5, & x_6, & y_1, & y_2 & \ge 0. \end{cases}$$

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- 1. is already solved for a basis, and
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$$\xi = -y_1 - y_2$$

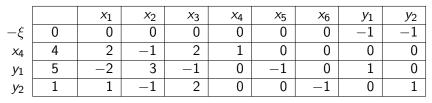
$$\begin{cases} 2x_1 - x_2 + 2x_3 + x_4 & = 4 \\ -2x_1 + 3x_2 - x_3 & -x_5 + y_1 & = 5 \\ x_1 - x_2 + 2x_3 & -x_6 + y_2 & = 1 \\ x_1, x_2, x_3, x_4, x_5, x_6, y_1, y_2 \ge 0. \end{cases}$$

$$x_7 = y_1 \text{ and } x_8 = y_2, \text{ bfs } (0, 0, 0, 4, 0, 0, 5, 1)^{T_{\text{constrain}}} = 0$$

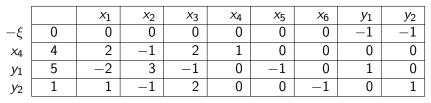
		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	<i>x</i> 6	<i>y</i> ₁	<i>y</i> ₂
$-\xi$	0	0	0	0	0	0	0	-1	-1
<i>X</i> 4	4	2	$^{-1}$	2	1	0	0	0	0
<i>y</i> 1	5	-2	3	-1	0	$^{-1}$	0	1	0
<i>y</i> ₂	1	1	-1	2	0	0	-1	0	1

Can we start pivoting?

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- Can we start pivoting?
- ► NO!, the coefficients of the basic variables y₁ and y₂ in row 0 are not zero.



- Can we start pivoting?
- ► NO!, the coefficients of the basic variables y₁ and y₂ in row 0 are not zero.
- Remove y_1 and y_2 from the top equation.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	x ₆	<i>y</i> ₁	<i>y</i> ₂			
$-\xi$	6	-1	2	1	0	-1	-1	0	0			
<i>X</i> 4	4	2	-1	2	1	0	0	0	0			
<i>y</i> 1	5	-2	3	$^{-1}$	0	$^{-1}$	0	1	0			
<i>y</i> 2												
bfs (bfs $(0, 0, 0, 4, 0, 0, 5, 1)$, basis $\{4, 7, 8\}$.											

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Pick a pivot column.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	x ₆	<i>y</i> ₁	<i>y</i> ₂			
$-\xi$	6	-1	2	1	0	-1	-1	0	0			
<i>x</i> 4	4	2	-1	2	1	0	0	0	0			
<i>y</i> 1	5	-2	3	$^{-1}$	0	$^{-1}$	0	1	0			
<i>y</i> ₂												
bfs (bfs $(0, 0, 0, 4, 0, 0, 5, 1)$, basis $\{4, 7, 8\}$.											

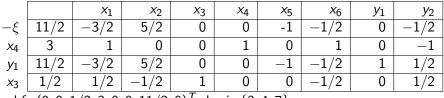
- Pick a pivot column.
- We pick 3. Column 2 would have also worked. What is the pivot row?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	x ₆	<i>y</i> ₁	<i>y</i> ₂			
$-\xi$	6	-1	2	1	0	-1	-1	0	0			
<i>x</i> 4	4	2	$^{-1}$	2	1	0	0	0	0			
<i>y</i> 1	5	-2	3	$^{-1}$	0	$^{-1}$	0	1	0			
<i>y</i> ₂												
bfs (bfs $(0, 0, 0, 4, 0, 0, 5, 1)$, basis $\{4, 7, 8\}$.											

- Pick a pivot column.
- We pick 3. Column 2 would have also worked. What is the pivot row?
- Row 3.What will the new basis be?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	x ₆	<i>y</i> ₁	<i>y</i> ₂			
$-\xi$	6	-1	2	1	0	-1	-1	0	0			
<i>X</i> 4	4	2	-1	2	1	0	0	0	0			
<i>y</i> 1	5	-2	3	$^{-1}$	0	$^{-1}$	0	1	0			
<i>y</i> 2												
bfs (bfs $(0, 0, 0, 4, 0, 0, 5, 1)$, basis $\{4, 7, 8\}$.											

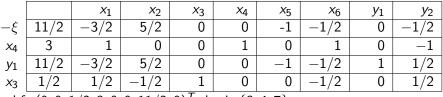
- Pick a pivot column.
- We pick 3. Column 2 would have also worked. What is the pivot row?
- Row 3.What will the new basis be?



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bfs $(0, 0, 1/2, 3, 0, 0, 11/2, 0)^T$, basis $\{3, 4, 7\}$.

Pick a pivot column.



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bfs $(0, 0, 1/2, 3, 0, 0, 11/2, 0)^T$, basis $\{3, 4, 7\}$.

- Pick a pivot column.
- Only option 2. What is the pivot row?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>X</i> 5	<i>x</i> 6	<i>y</i> ₁	<i>y</i> ₂
$-\xi$	11/2	-3/2	5/2	0	0	-1	-1/2	0	-1/2
<i>x</i> 4	3	1	0	0	1	0	1	0	-1
<i>y</i> ₁	11/2	-3/2	5/2	0	0	$^{-1}$	-1/2	1	1/2
<i>x</i> 3	1/2	1/2	-1/2	1	0	0	-1/2	0	1/2

bfs $(0, 0, 1/2, 3, 0, 0, 11/2, 0)^T$, basis $\{3, 4, 7\}$.

- Pick a pivot column.
- Only option 2. What is the pivot row?
- Row 2. What will the new basis be?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	<i>x</i> 6	<i>y</i> ₁	<i>y</i> ₂
$-\xi$	11/2	-3/2	5/2	0	0	-1	-1/2	0	-1/2
<i>x</i> 4	3	1	0	0	1	0	1	0	-1
<i>y</i> ₁	11/2	-3/2	5/2	0	0	$^{-1}$	-1/2	1	1/2
<i>x</i> 3	1/2	1/2	-1/2	1	0	0	-1/2	0	1/2

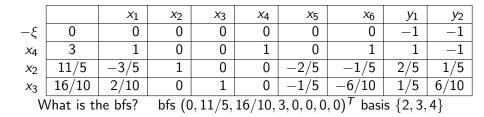
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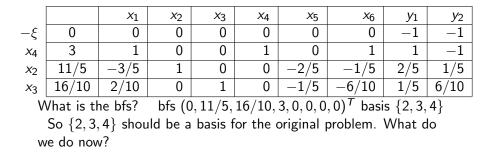
bfs $(0, 0, 1/2, 3, 0, 0, 11/2, 0)^T$, basis $\{3, 4, 7\}$.

- Pick a pivot column.
- Only option 2. What is the pivot row?
- Row 2. What will the new basis be?
- ► {2,3,4}.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>X</i> 4	<i>X</i> 5	<i>x</i> 6	<i>y</i> 1	<i>y</i> ₂
$-\xi$	0	0	0	0	0	0	0	-1	-1
<i>x</i> 4	3	1	0	0	1	0	1	1	$^{-1}$
<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5	2/5	1/5
<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10	1/5	6/10
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What is the bfs?





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		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>X</i> 4	<i>X</i> 5	<i>x</i> 6	<i>y</i> 1	<i>y</i> 2			
$-\xi$	0	0	0	0	0	0	0	-1	-1			
<i>x</i> 4	3 1 0 0 1 0 1 1											
<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5	2/5	1/5			
<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10	1/5	6/10			
V	What is the bfs? bfs $(0, 11/5, 16/10, 3, 0, 0, 0, 0)^T$ basis $\{2, 3, 4\}$											
	So $\{2,3,4\}$ should be a basis for the original problem. What do											
W	we do now? Remove columns corresponding to the auxilliary											
Vä	variables (i.e. the variables y_1, y_2) from the tableau.											

			<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	<i>x</i> 6
	-z	0	1	$^{-1}$	1	0	0	0
	<i>X</i> 4	3	1	0	0	1	0	1
	<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5
	<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10
~	10 1			$a \rightarrow T$		(0.0.1	<u>,</u>	

What do we do now? Can we pivot?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>X</i> 5	<i>x</i> 6
-z	0	1	$^{-1}$	1	0	0	0
<i>x</i> 4	3	1	0	0	1	0	1
<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5
<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10

- What do we do now? Can we pivot?
- Remove basic variables x₂ and x₃ from Row 0

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> 4	<i>x</i> 5	<i>x</i> 6
- <i>z</i>	3/5	1/5	0	0	0	-1/5	2/5
<i>X</i> 4	3	1	0	0	1	0	1
<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5
<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10

What is the pivot row and what is the pivot column?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>x</i> 5	<i>x</i> ₆
-z	3/5	1/5	0	0	0	-1/5	2/5
<i>X</i> 4	3	1	0	0	1	0	1
<i>x</i> ₂	11/5	-3/5	1	0	0	-2/5	-1/5
<i>x</i> 3	16/10	2/10	0	1	0	-1/5	-6/10

- What is the pivot row and what is the pivot column?
- Pick 6 as the pivot column and 1 is the pivot row.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>X</i> 5	<i>x</i> 6
-z	-3/5	-1/5	0	0	-2/5	-1/5	0
<i>x</i> 6	3	1	0	0	1	0	1
<i>x</i> ₂	14/5	-2/5	1	0	1/5	-2/5	0
<i>x</i> 3	17/5	4/5	0	1	3/5	-1/5	0

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► This tableau corresponds to the basic solution (0, 14/5, 17/5, 0, 0, 3) which gives -z = 3/5.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>X</i> 5	<i>x</i> 6
-z	-3/5	-1/5	0	0	-2/5	-1/5	0
<i>x</i> 6	3	1	0	0	1	0	1
<i>x</i> ₂	14/5	-2/5	1	0	1/5	-2/5	0
<i>x</i> 3	17/5	4/5	0	1	3/5	-1/5	0

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► This tableau corresponds to the basic solution (0, 14/5, 17/5, 0, 0, 3) which gives -z = 3/5.

Is this solution optimal?

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>X</i> 5	<i>x</i> ₆
-z	-3/5	-1/5	0	0	-2/5	-1/5	0
<i>x</i> 6	3	1	0	0	1	0	1
<i>x</i> ₂	14/5	-2/5	1	0	1/5	-2/5	0
<i>x</i> 3	17/5	4/5	0	1	3/5	-1/5	0

- ► This tableau corresponds to the basic solution (0, 14/5, 17/5, 0, 0, 3) which gives -z = 3/5.
- Is this solution optimal?
- > Yes, because we do not have negative entries in Row 0.

		<i>x</i> ₁	<i>x</i> ₂	<i>x</i> 3	<i>x</i> ₄	<i>x</i> 5	<i>x</i> ₆
-z	-3/5	-1/5	0	0	-2/5	-1/5	0
<i>x</i> 6	3	1	0	0	1	0	1
<i>x</i> ₂	14/5	-2/5	1	0	1/5	-2/5	0
<i>x</i> 3	17/5	4/5	0	1	3/5	-1/5	0

- ► This tableau corresponds to the basic solution (0, 14/5, 17/5, 0, 0, 3) which gives -z = 3/5.
- Is this solution optimal?
- Yes, because we do not have negative entries in Row 0.
- ► The optimal value is -3/5 at the optimal basic feasible solution (0, 14/5, 17/5, 0, 0, 3)^T