Test 4 topics

If a theorem is not followed by "(with proof)", then you are not responsible for the proof, you are only responsible for the statement of the theorem.

About one half of the material for test 4 will be selected from the following topics

(1) Theorem: max flow equals min cut
(2) Ellipsoid method (section 7.1)
   (a) definitions: input size, size of a integer, rational number, vector, matrix and linear program (pg 106-107)
   (b) definition: polynomial algorithm for linear programming (pg 107)
   (c) definition: Ellipsoid, affine map, positive definite matrix, equation 7.1 (pg 109)
   (d) Four steps of algorithm for softened version of problem (pg 110)
   (e) Statement of E1 and E2 - (you do **not** need to know that $K = 2^{\phi}$, $\eta = 2^{-5\phi}$ or that $\varepsilon = 2^{-6\phi}$)
(3) Definition of an approximation algorithm and the approximation factor (pg 151)
(4) Approximation algorithm for vertex cover (section 3.3)
(5) Maximum Independent set as a integer program (section 3.4)
(6) Approximation algorithm for the machine scheduling problem (section 8.3) (the handout `http://www.math.uiuc.edu/~molla/2015_spring_math482/scheduling.pdf` describes the algorithm given in the book)
   (a) Definition of makespan (page 149)
   (b) Linear programming formulation of problem
   (c) Definition of LPR(T)
   (d) Lemma 8.3.2
   (e) Lemma 8.3.3
   (f) Theorem 8.3.4 (with proof)
(7) Satisfiability problem and its formulation as an integer program
(8) Formulating problems as integer programs (similar to questions 4 and 5 in homework 9 and question 3 on homework 10)

About one half of the material for test 4 will be selected from the following older topics

(1) Revised simplex (see Revised simplex example)
   (a) the CARRY matrix
   (b) constructing the initial carry matrix for 2-phase simplex and regular primal simplex
   (c) Computing relative cost: $\overline{c}_j$ during primal simplex and second phase of two phase simplex and $\overline{d}_j$ during first phase of two phase simplex.
   (d) moving from one CARRY matrix to the next
   (e) moving from first phase of 2-phase simplex to second phase when doing revised simplex
(2) Integer programing definition
(3) LP Relaxation of an integer program
(4) Definition: Incidence matrices of graphs and digraphs
(5) Maximum matching and minimum vertex cover using integer linear programming

(6) König's Theorem and Hall's Theorem
(7) Definition of totally unimodular matrices (8.2.1)
(8) Lemma 8.2.4
(9) Incidence matrices of bipartite graph and directed graphs are totally unimodular
(10) Dualization recipe
(11) Simplex method and two phase simplex method (see examples on website)
(12) Tableau $\mathcal{T}(B)$, matrix tableau and interpreting the tableau
(13) 6.1.1 (Weak Duality) (with proof)
(14) Theorem (Duality Theorem of Linear Programming/Strong Duality)
(15) Farkas Lemma (6.4.1)
(16) Dual simplex method
(17) Complementary slackness (with proof)
(18) Theorem 4.2.1
(19) Theorem 4.2.2 (with proof)
(20) Theorem 4.2.3
(21) Understand examples 2.1, 2.2, 2.3 and 2.4 from book - converting a word problem into a LP
(22) Equational/Standard form - converted to standard form
(23) Notation: A and $m \times n$ matrix, $\mathbf{x}$ and n-dimensional vector, $B \subseteq [n] := \{1, \ldots, n\}$, what is A_B and $\mathbf{x}_n$?
(24) Convert any LP to the form

$$\max \mathbf{c}^T\mathbf{x} \text{ subject to } A\mathbf{x} \leq \mathbf{b}$$

(25) Def: basic feasible solution, basic variables, nonbasic variables, basis, feasible basis, degenerate basic feasible solution, degenerate linear program
(26) Definition: feasible solution
(27) Definition: objective function
(28) Definition: convex polyhedron
(29) Definition: optimal solution, optimum
(30) Definition: constraint, system of linear equation/inequalities