

## Test 3 topics

Test 3 will focus on the revised simplex and primal dual simplex

If a theorem is not followed by “(with proof)”, then you are not responsible for the proof you are only responsible for the statement of the theorem. The best way to prepare is to review old homework and quizzes.

- (1) Revised simplex (see Revised simplex example)
  - (a) the CARRY matrix
  - (b) constructing the initial carry matrix for 2-phase simplex and regular primal simplex
  - (c) Computing relative cost:  $\bar{c}_j$  during primal simplex and second phase of two phase simplex and  $\bar{d}_j$  during first phase of two phase simplex.
  - (d) moving from one CARRY matrix to the next
  - (e) moving from first phase of 2-phase simplex to second phase when doing revised simplex
- (2) Revised simplex and max flow (see Max flow and revised simplex)
  - (a) arc-chain incidence matrix  $D$
  - (b) max flow LP using arc-chain incidence matrix
  - (c) extracting  $\pi^T$  from the tableau
  - (d) finding a profitable path/column to bring into the basis by finding a shortest path in  $G$  with weight from  $\pi$  on the edges
- (3) Primal dual method (see Primal dual simplex example and Primal dual simplex notes)
  - (a) The relationship between primal dual simplex and complementary slackness
  - (b) Computing  $J$  given a feasible solution to the dual
  - (c) What does it mean when the optimal value of restricted primal is 0 and not equal to 0?
  - (d) What does it mean when the optimal value of dual of the restricted primal is 0 and not equal to 0?
  - (e) computing  $\theta$  given a feasible solution to the dual and a feasible solution to the dual of the restricted primal
  - (f) moving from one iteration to the next
- (4) Primal dual and min cost path (see Primal dual and shortest path)
  - (a) Formulation of min cost problem for primal dual
  - (b) dual of min cost problem
  - (c) computing  $J$  given a feasible solution to the dual
  - (d) dual of the restricted primal
  - (e) computing  $\theta$  given a feasible solution to the dual and a feasible solution to the dual of the restricted primal
  - (f) the set  $W$  and  $\bar{W}$
  - (g) the algorithm for finding a shortest path
- (5) Primal dual and max flow Primal dual and max flow)
  - (a) Formulation of max flow problem as the dual to a minimization problem in standard form
  - (b) The value of a flow  $|f|$ .

- (c) Dual of the restricted primal as a min cost path problem
- (d)  $f$ -augmenting  $s, t$ -path and its relationship with the dual of the restricted primal problem
- (e) computing  $\theta$  given a feasible solution to the dual and a feasible solution to the dual of the restricted primal
- (f) the augmenting path algorithm for finding a max flow

From test 1 and test 2

- (1) Dual simplex method
- (2) Complementary slackness (with proof)
- (3) Farkas Lemma (6.4.1)
- (4) Variants of Farkas Lemma (6.4.3)
- (5) Zero-sum games (Minimax theorem 8.1.3 and associated terms)
- (6) How to solve a matrix game using linear programming
- (7) Integer programming definition
- (8) LP Relaxation of an integer program
- (9) Definition of graphs and digraphs and their incidence matrices
- (10) Definition of totally unimodular matrices
- (11) Incidence matrices of bipartite graph and directed graphs are totally unimodular
- (12) Maximum matching and minimum vertex cover using integer linear programming
- (13) König's Theorem and Hall's Theorem
- (14) Solving a 2d LP graphically
- (15) Definition: feasible solution
- (16) Definition: object function
- (17) Definition: convex polyhedron
- (18) Definition: optimal solution, optimum
- (19) Definition: constraint, system of linear equation/inequalities
- (20) Convert any LP to the form

$$\max \mathbf{c}^T \mathbf{x} \text{ subject to } A\mathbf{x} \leq \mathbf{b}$$

- (21) Understand examples 2.1, 2.2, 2.3 and 2.4 from book - converting a word problem into a LP
- (22) Equational/Standard form - converted to standard form
- (23) Notation:  $A$  and  $m \times n$  matrix,  $\mathbf{x}$  and  $n$ -dimensional vector,  $B \subseteq [n] := \{1, \dots, n\}$ , what is  $A_B$  and  $\mathbf{x}_n$ ?
- (24) Def: basic feasible solution, basic variables, nonbasic variables, basis, feasible basis, degenerate basic feasible solution, degenerate linear program
- (25) Theorem 4.2.1
- (26) Theorem 4.2.2 (with proof)
- (27) Theorem 4.2.3
- (28) Simplex method and two phase simplex method (see examples on website)
- (29) Tableau  $\mathcal{T}(B)$ , matrix tableau and interpreting the tableau
- (30) Theorem 5.5.1 (you do not need to memorize the formulas)
- (31) pivot rules - lexicographic, Bland's pivot rule, largest coefficient (Dantzig's original rule)

- (32) You must know that lexicographic and Bland's rule do not cycle (the proof is not required)
- (33) Definition: lexicographic ordering of vectors
- (34) Lemma (not in book) If two different basis  $B$  and  $B'$  correspond to the same basic feasible solution  $\mathbf{x}$  then  $\mathbf{x}$  is degenerate. (with proof)
- (35) Lemma (not in book) Once the simplex method enters a cycle the basic feasible solution is fixed and this basic feasible solution is degenerate.
- (36) Theorem (not in book) If an LP is not degenerate, then the simplex method will never cycle
- (37) 6.1.1 (Weak Duality) (with proof)
- (38) Canonical form (not described as Canonical form in the book)

$$\max \mathbf{c}^T \mathbf{x} \text{ subject to } A\mathbf{x} \leq b, \mathbf{x} \geq \mathbf{0}$$

- (39) Dualization recipe
- (40) Theorem (Duality Theorem of Linear Programming/Strong Duality)