

Test 2 topics

If a theorem is not followed by “(with proof)”, then you are not responsible for the proof you are only responsible for the statement of the theorem. The best way to prepare is to review old homework and quizzes.

New material

- (1) Dual simplex method
- (2) Complementary slackness (with proof)
- (3) Farkas Lemma (6.4.1)
- (4) Variants of Farkas Lemma (6.4.3)
- (5) Zero-sum games (Minimax theorem 8.1.3 and associated terms)
- (6) How to solve a matrix game using linear programming
- (7) Integer programming definition
- (8) Definition of graphs and digraphs and their incidence matrices
- (9) Definition of totally unimodular matrices
- (10) Incidence matrices of bipartite graph and directed graphs are totally unimodular
- (11) Maximum matching and minimum vertex cover using integer linear programming
- (12) König’s Theorem and Hall’s Theorem

From test 1

- (1) Solving a 2d LP graphically
- (2) Definition: feasible solution
- (3) Definition: object function
- (4) Definition: convex polyhedron
- (5) Definition: optimal solution, optimum
- (6) Definition: constraint, system of linear equation/inequalities
- (7) Convert any LP to the form

$$\max \mathbf{c}^T \mathbf{x} \text{ subject to } A\mathbf{x} = \mathbf{b}$$

- (8) Understand examples 2.1, 2.2, 2.3 and 2.4 from book - converting a word problem into a LP
- (9) Equational/Standard form - converted to standard form
- (10) Notation: A and $m \times n$ matrix, \mathbf{x} and n -dimensional vector, $B \subseteq [n] := \{1, \dots, n\}$, what is A_B and \mathbf{x}_n ?
- (11) Def: basic feasible solution, basic variables, nonbasic variables, basis, feasible basis, degenerate basic feasible solution, degenerate linear program
- (12) Theorem 4.2.1
- (13) Theorem 4.2.2 (with proof)
- (14) Theorem 4.2.3
- (15) Simplex method and two phase simplex method (see examples on website)
- (16) Tableau $\mathcal{T}(B)$, matrix tableau and interpreting the tableau
- (17) Theorem 5.5.1 (you do not need to memorize the formulas)
- (18) pivot rules - lexicographic, Bland’s pivot rule, largest coefficient (Dantzig’s original rule)
- (19) You must know that lexicographic and Bland’s rule do not cycle (the proof is not required)

- (20) Definition: lexicographic ordering of vectors
- (21) Lemma (not in book) If two different basis B and B' correspond to the same basic feasible solution \mathbf{x} then \mathbf{x} is degenerate. (with proof)
- (22) Lemma (not in book) Once the simplex method enters a cycle the basic feasible solution is fixed and this basic feasible solution is degenerate.
- (23) Theorem (not in book) If an LP is not degenerate, then the simplex method will never cycle
- (24) 6.1.1 (Weak Duality) (with proof)
- (25) Canonical form (not described as Canonical form in the book)

$$\max \mathbf{c}^T \mathbf{x} \text{ subject to } A\mathbf{x} \leq b, \mathbf{x} \geq \mathbf{0}$$

- (26) Dualization recipe
- (27) Theorem (Duality Theorem of Linear Programming/Strong Duality)