Math 482 **HW10** Name: _____

Due Friday, May 1, 2015

1. (7 points) Suppose we are to make a schedule for the jobs $J := \{5, 6, 7, 8, 9, 10, 11\}$ on the machines $M := \{1, 2, 3, 4\}$. As in lecture and in section 8.3 of the book, let LPR(6) be the linear program relaxation of the machine scheduling integer program with the additional condition that if job j takes longer than 6 units to complete on a machine i, then job j cannot be scheduled on machine i. Suppose that (t^*, x^*) is an optimal basic feasible solution of LPR(6) and $t^* = 8$. Also, suppose that the non-zero entries of x^* are $x^*_{1,5}, x^*_{1,7}, x^*_{1,11}, x^*_{2,5}, x^*_{2,6}, x^*_{2,9}, x^*_{3,10}, x^*_{3,11}, x^*_{4,5}, x^*_{4,8}$. Use the approximate algorithm discussed in class and in section 8.3 of the book to construct an integer schedule with makespan at most 14.

2. (7 points) Consider the Boolean formula

$$(x_1 + x_2) \cdot (x_1 + \overline{x}_2) \cdot (\overline{x}_1 + x_2) \cdot (\overline{x}_1 + \overline{x_2} + x_3) \cdot (\overline{x}_1 + \overline{x_3})$$

(As we discussed in class, $+$ corresponds to "or" and $\cdot$ corresponds to "and" and $\overline{x}$ is the negation of the boolean variable x).

Write an integer program that has a feasible solution if and only if the formula is satisfiable. Is the formula satisfiable (for example the formula $x_1 \cdot (\overline{x}_2 + \overline{x}_1)$ is satisfiable with the truth assignment $x_1 = 1$, $\overline{x}_2 = 1$)? Does the LP relaxation of the integer program have a feasible solution?

3. (6 points) Suppose that you are interested in choosing a set 7 investments and c_i is the expected return from investment i for every $i \in [7]$. Formulate a integer program in which the variables can only take on the values 0 or 1 that maximizes the expected return and that satisfies the following constraints:

    1. You cannot invest in all 7 of the possible investments.
    2. Investment 1 cannot be chosen if investment 3 has been chosen.
    3. Investment 4 can be chosen only if investment 2 is also chosen.
    4. You must choose either at least one of the investment 1,2 or 3 or at least two of the investments 2, 4, 5 and 6.