Due Friday, April 10, 2015

1.

$$\text{Maximize } z = -4x_1 - x_2 - 3x_3 - x_4$$

subject to

$$
\begin{aligned}
x_1 + x_2 + x_3 \quad\quad\ &= \ 3, \\
x_1 \quad\ + x_3 + 3x_4 &= \ 2, \\
x_2 + x_3 + x_4 &= \ 2, \\
x_1, x_2, x_3, x_4 &\geq \ 0.
\end{aligned}
$$

(a) Solve the restricted primal that corresponds to the dual feasible vector $y^T = [0, -1/3, 0]$ using the simplex method.

(b) Determine if the original dual solution was optimal and extract a feasible solution for the dual of the restricted primal from the final tableau of the restricted primal problem.

(c) If the original solution was not optimal, use the primal dual method to calculate a new feasible solution to the dual and write the restricted primal problem that corresponds to this new solution.

2. Beginning with the vector $\pi = (0,0,0,0,0,0)^T$ which is feasible for the dual, use the primal dual method as described in class and in the notes to find the shortest path from s to t in the weighted graph shown below. For each iteration, you must write π, J, π^r (π^r is the dual of the restricted primal which is the same as the variable $\bar\pi$ we used in class) and draw the edges corresponding to J. The worksheet provided may be helpful

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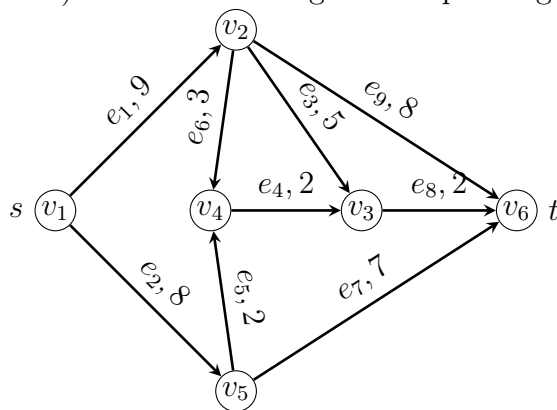
3. Apply the Ford-Fulkerson algorithm to the following network. On each iteration, list the edges of the augmenting path and θ. Prove that the flow is maximal by finding a minimal cut. The worksheet provided may be helpful.
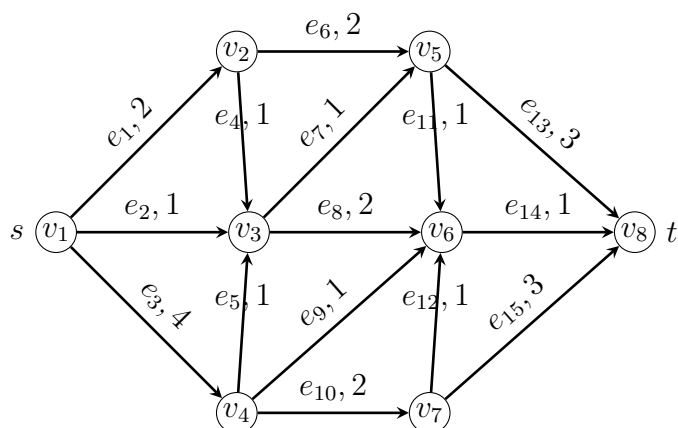
4. Suppose there are n men and n women and m marriage brokers (labeled $c_1, \ldots, c_m$). Each broker has a list of men and women as clients and can arrange marriages between any pairs of men and women on the list. In addition, we restrict the number of marriages that broker i can arrange to a maximum of b_i. Each man can be married to at most one women and each women can be married to at most one man. Translate the problem of finding a solution with the most marriages into one of finding the maximum flow in a flow network.