Math 482 **HW6** Name: ____________________

Due Friday, March 13, 2015

1. (5 points) Use the complementary slackness condition to check whether the vector $[3,\ -1,\ 0,\ 2]^T$ is an optimal solution to the problem

$$\text{Maximize } z = 6x_1 + x_2 - x_3 - x_4$$

subject to
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 & \leq & 5, \\ 3x_1 + x_2 - x_3 & \leq & 8, \\ x_2 + x_3 + x_4 & = & 1, \\ x_3, x_4 & \geq & 0. \end{cases}$$

2. (5 points) Let A be an $m \times n$ matrix of rank m and let P be the following LP in standard form.

$$\text{Minimize } z = c^T x$$
$$\text{subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

Prove that if the P has an optimal solution and P has no degenerate optimal solutions, then there is a **unique** optimal solution to the dual of P. (Hint: Use the complementary slackness condition and the fact that if an LP in standard form has an optimal solution, then it has an optimal basic feasible solution)

3. (5 points) Prove that if A is TUM, then the matrices A^T, $-A$ and $(A|A)$ are all TUM.

4. (5 points) Show that every matrix with entries $-1, 0$ and 1 such that at most one row has more than one non-zero entry is totally unimodular. Give an example of a totally unimodular matrix with at least three non-zero elements in every column and in every row.