

Due Friday, March 6, 2015

1. (5 points) Use the dual simplex method to find an optimal solution to the problem

$$\text{Maximize } z = -7x_1 - x_2 - 3x_3 - x_4$$

subject to

$$\begin{cases} 2x_1 - 3x_2 - x_3 + x_4 & \geq 8, \\ 6x_1 + x_2 + 2x_3 - 2x_4 & \geq 12, \\ -x_1 + x_2 + x_3 + x_4 & \geq 10, \\ x_1, x_2, x_3, x_4 & \geq 0. \end{cases}$$

2. (5 points) The additional constraints

$$\begin{aligned} x_1 + 5x_2 + x_3 + 7x_4 &\leq 50, \\ 3x_1 + 2x_2 - 2x_3 - x_4 &\leq 20 \end{aligned}$$

are added to those of Problem 3. Solve the new problem starting from the optimal tableau for Problem 3.

3. (5 points) Let the matrix define a game and let Alice be the player whose pure strategies are represented by the rows of the matrix and Bob be the player whose pure strategies are represented by the columns of the matrix. What is the optimal pure strategy for Alice and what is expected payout given that choice? In other words, if Alice must play the same pure strategy in every turn of the game, what pure strategy should she play and what is the expected payout assuming Bob plays optimally? Determine the same information for Bob.

$$\begin{pmatrix} 4 & 8 & 1 & 2 & 6 & 7 \\ 8 & 2 & 3 & 5 & 9 & 2 \\ 1 & 8 & 1 & 7 & 3 & 3 \\ 6 & 5 & 6 & 8 & 6 & 3 \\ 2 & 6 & 2 & 4 & 7 & 1 \\ 3 & 9 & 1 & 5 & 2 & 6 \end{pmatrix}.$$

4. (5 points) Solve the game with the payoff matrix

$$\begin{pmatrix} 3 & 5 & 3 & -2 & 0 \\ 3 & 7 & 3 & -1 & 1 \\ 2 & -4 & 1 & 3 & 4 \\ 1 & -5 & 1 & 3 & 0 \end{pmatrix}$$

Here both players are allowed to use mixed strategies. You must write the linear program that you must solve, but you can use a linear program solver of your choice to actually solve the linear program (for example you can use <http://www.wolframalpha.com>).