Math 482 **HW2** **1/30/2015** Name: ____________________

Due Friday, February 6, 2015

1. Find all of the basic feasible solutions of the following LP, the associated value of each basic feasible solution and the associated basis for each basic feasible solution. From this, determine an optimal solution and optimal value for the LP. **Do not use the simplex method on this problem**

$$z = 4x_1 + 2x_2 + x_3 \quad \longrightarrow \quad \max$$

subject to

$$\begin{cases} 2x_1 + & 6x_2 + & 2x_3 & = 10 \\ x_1 + & 2x_2 + & 3x_3 & = 6 \\ x_1, & x_2, & x_3 & \geq 0. \end{cases}$$

2. A farmer has 100 acres of land which he has decided to make part arable and part grass. Part of the land may also lie fallow. He can make an annual profit of \$ 150/acre from arable land and \$ 100/acre from grass. Each year arable land requires 25 hours work per acre, and grass requires 10 hours work per acre. The farmer does not want to work more than 2000 hours in any year. How should he divide his land between arable and grass so as to maximize his annual profit? *You do not need to use the simplex method to solve this problem, but you can if you want to.*

3. We are given two instances of LP in standard form:

$$\text{Maximize } z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to
$$\begin{cases} \mathbf{Ax} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0} \end{cases}$$

and

$$\text{Maximize } \xi = -c_1x_1 - c_2x_2 - \ldots - c_nx_n$$

subject to
$$\begin{cases} \mathbf{Ax} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0}. \end{cases}$$

The numbers $c_1, \ldots, c_n$, matrix $\mathbf{A}$, and vector $\mathbf{b}$ are the same in both instances. Can both instances have feasible solutions with arbitrarily large cost? In 'yes', give an example; if 'not', prove so.

4. Use the simplex method to solve the LP

$$\text{Maximize } z = 5x_1 + 5x_2 + 3x_3$$

subject to
$$\begin{cases} \begin{array}{rrrcl} x_1 & +3x_2 & +x_3 & \leq & 3 \\ -x_1 & & +3x_3 & \leq & 4 \\ 2x_1 & -2x_2 & +2x_3 & \leq & 4 \\ 2x_1 & +3x_2 & -x_3 & \leq & 2 \\ & & x_1, x_2, x_3 & \geq & 0. \end{array} \end{cases}$$

5. Prove that if variable x_s is moved out of the basis of a linear program at some step of the simplex method, then at the next step it will NOT be moved back into the basis.