Example of cycling with simplex

If the tableau T_1 is reached, then the simplex method will cycle when the following pivot rules are used: Pick the pivot column by selecting the most largest positive entry in row 0. If there are ties when picking the pivot row, pick the row that will cause the variable with the smallest index to leave the basis. Note that the basic feasible solution corresponding to every tableau is the same and that $T_1 = T_7$. The pivot entries are circled.

$$T_1 = \left(\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -3 & \frac{3}{4} & -20 & \frac{1}{2} & -6 & 0 & 0 & 0 \\ \hline 0 & \mathbf{\frac{1}{4}} & -8 & -1 & 9 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

basic feasible solution $x^{(1)} = (0,0,0,0,0,1)$, basis $B^{(1)} = \{5,6,7\}$.

$$T_2 = \left(\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -3 & 0 & 4 & \frac{7}{2} & -33 & -3 & 0 & 0 \\ \hline 0 & 1 & -32 & -4 & 36 & 4 & 0 & 0 \\ 0 & 0 & \mathbf{4} & \frac{3}{2} & -15 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

basic feasible solution $x^{(2)} = (0,0,0,0,0,1)$, basis $B^{(2)} = \{1,6,7\}$.

$$T_3 = \left(\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -3 & 0 & 0 & 2 & -18 & -1 & -1 & 0 \\ \hline 0 & 1 & 0 & \mathbf{8} & -84 & -12 & 8 & 0 \\ 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

basic feasible solution $x^{(3)} = (0,0,0,0,0,1)$, basis $B^{(3)} = \{1,2,7\}$.

$$T_4 = \left(\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -3 & -\frac{1}{4} & 0 & 0 & 3 & 2 & -3 & 0 \\ \hline 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & -\frac{3}{64} & 1 & 0 & \mathbf{\frac{3}{16}} & \frac{1}{16} & -\frac{1}{8} & 0 \\ 1 & -\frac{1}{8} & 0 & 0 & 21/2 & \frac{3}{2} & -1 & 1 \end{array}\right)$$

basic feasible solution $x^{(4)} = (0,0,0,0,0,1)$, basis $B^{(4)} = \{3,2,7\}$.

$$T_5 = \begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \end{array}$$
$$\left(\begin{array}{c|ccccccc} -3 & \frac{1}{2} & -16 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & -\frac{5}{2} & 56 & 1 & 0 & \mathbf{2} & -6 & 0 \\ 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\ 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1 \end{array}\right)$$

basic feasible solution $x^{(5)} = (0,0,0,0,0,1)$, basis $B^{(5)} = \{3,4,7\}$.

$$T_6 = \begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \end{array}$$
$$\left(\begin{array}{c|ccccccc} -3 & \frac{7}{4} & -44 & -\frac{1}{2} & 0 & 0 & 2 & 0 \\ \hline 0 & -\frac{5}{4} & 28 & \frac{1}{2} & 0 & 1 & -3 & 0 \\ 0 & \frac{1}{6} & -4 & -\frac{1}{6} & 1 & 0 & \mathbf{\frac{1}{3}} & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

basic feasible solution $x^{(6)} = (0,0,0,0,0,1)$, basis $B^{(6)} = \{5,4,7\}$.

$$T_7 = \begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \end{array}$$
$$\left(\begin{array}{c|ccccccc} -3 & \frac{3}{4} & -20 & \frac{1}{2} & -6 & 0 & 0 & 0 \\ \hline 0 & \mathbf{\frac{1}{4}} & -8 & -1 & 9 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

basic feasible solution $x^{(7)} = (0,0,0,0,0,1)$, basis $B^{(7)} = \{5,6,7\}$.