Math 482 **HW11** Section: _____ Name: _____

Finish before the final

Students in section D13 (three credit hours) need to solve Any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Solve the INTEGER linear program below using the branch and bound algorithm.

$$\text{Minimize } z = -x_1 - x_2$$

subject to

$$\begin{cases} 4x_1 + x_2 & \leq & 16, \\ -2x_1 + 5x_2 & \leq & 10, \\ x_1, x_2 & \geq & 0, \\ x_1 \in \mathbb{Z}, & x_2 \in \mathbb{Z} & \end{cases}$$

Draw the feasible regions of all feasible nodes in the plane $0x_1x_2$.

2. Consider the following integer program:

$$\begin{array}{lllll} \text{Minimize} & -10x_1 & -8x_2 & & \\ \text{subject to} & 11x_1 & +7x_2 & \leq 38 \\ & 7x_1 & +9x_2 & \leq 35 \\ & x_1, & x_2 & \geq 0 \\ & x_1, & x_2 & \in \mathbb{Z}, \end{array}$$

and let P be its integer relaxation. Suppose that while solving this program with the branch and bound method you encounter the following nodes:

Node name	Added constraint	Solution	
P :		$x^* = (97/50, 119/50)^T$,	$z^* = -1922/50 \approx -38.4$
PA :	$x_1 \geq 2$,	$x^* = (2, 16/7)^T$,	$z^* = -268/7 \approx -38.3$
PAA :	$x_2 \geq 3$,	infeasible	
PAB :	$x_2 \leq 2$,	$x^* = (24/11, 2)^T$,	$z^* = -416/11 \approx -37.8$
$PABA$:	$x_1 \geq 3$,	$x^* = (3, 5/7)^T$,	$z^* = -250/7 \approx -35.7$
$PABAA$:	$x_2 \geq 1$,	infeasible	
$PABAB$:	$x_2 \leq 0$,	$x^* = (38/11, 0)$,	$z^* = -380/11 \approx -34.5$
$PABABA$:	$x_1 \geq 4$,	infeasible	
$PABABB$:	$x_1 \leq 3$,	$x^* = (3, 0)$,	$z^* = -30$
$PABB$:	$x_1 \leq 2$,	$x^* = (2, 2)^T$,	$z^* = -36$
PB :	$x_1 \leq 1$,	$x^* = (1, 28/9)^T$,	$z^* = -314/9 \approx -34.9$
PBA :	$x_2 \geq 4$,	infeasible	
PBB :	$x_2 \leq 3$,	$x^* = (1, 3)^T$,	$z^* = -34$

What is the optimal solution of P? List the nodes that could kill other nodes and, for each such node, list the nodes that it kills. If a node kills another node that has descendants, you do not need to list the descendants only the parent node.