Math 482 **HW10** Section: ______________________ Name: ______________________

Due Friday, December 4, 2015

Students in section D13 (three credit hours) need to solve Any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Consider the Boolean formula
$$(x_1 + x_2) \cdot (x_1 + \overline{x}_2) \cdot (\overline{x}_1 + x_2) \cdot (\overline{x}_1 + \overline{x_2} + x_3) \cdot (\overline{x}_1 + \overline{x_3})$$
(As we discussed in class, + corresponds to "or" and $\cdot$ corresponds to "and" and $\overline{x}$ is the negation of the boolean variable x).

   Write an integer program that has a feasible solution if and only if the formula is satisfiable. Is the formula satisfiable (for example the formula $x_1 \cdot (\overline{x}_2 + \overline{x}_1)$ is satisfiable with the truth assignment $x_1 = 1$, $\overline{x}_2 = 1$)? Does the LP relaxation of the integer program have a feasible solution?

2. Consider the following integer program P:
$$\begin{array}{lccccl} & z = x_1 & & \to & \text{min} & \\ \text{subject to} & 3x_1 & -100x_2 & \geq & 1 \\ & 3x_1 & -101x_2 & \leq & 1 \\ & x_1, & x_2 & \geq & 0 \\ & x_1, & x_2 & & \text{integer} \end{array}$$
Solve the linear programming relaxation of P, obtaining an optimal solution x^* with cost z^* (You can solve the linear programming relaxation in any manner that you wish.) Obtain an integer vector x from x^* by rounding each component to the nearest integer. Is x an optimal solution to the integer program P? If it is not, find an optimal solution to the integer program P.

3. Suppose that you are interested in choosing a set 7 investments and c_i is the expected return from investment i for every $i \in [7]$. Formulate a integer program in which the variables can only take on the values 0 or 1 that maximizes the expected return and that satisfies the following constraints:
   1. You cannot invest in all 7 of the possible investments.
   2. Investment 1 cannot be chosen if investment 3 has been chosen.
   3. Investment 4 can be chosen only if investment 2 is also chosen.
   4. You must choose either at least one of the investment 1,2 or 3 or at least two of the investments 2, 4, 5 and 6.

   Hint: It might be helpful to add two additional variables, i.e. instead of 7 *zero-one-variables for the* 7 *investments, define* 9 *zero-one variables.*

4. Prove that if A is TUM, then the matrices A^T, $-A$ and $(A|A)$ are all TUM.

5. Show that every matrix with entries $-1, 0$ and 1 such that at most one row has more than one non-zero entry is totally unimodular. Give an example of a totally unimodular matrix with at least three non-zero elements in every column and in every row.