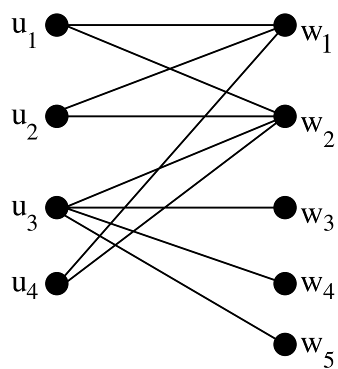
Math 482 **HW9** Section: ____________________ Name: ______________________________

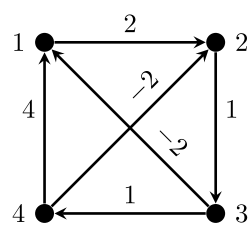
Due Friday, November 20, 2015

Students in section D13 (three credit hours) need to solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. A matching is a set of edges such that no vertex is contained in more than one edge. Using maximum flows, find a maximum matching in the bipartite graph below on the left. Prove that the matching is optimal. *Hint: Construct an flow network and then find a flow in this network that corresponds to a maximum matching in the graph. Show this flow is maximum by finding a minimum cut in the network. You should* ***clearly*** *write the flow network you are using, the max flow and the minimum cut in this network.*



2. Find the length of a shortest paths from every vertex to all other vertices in the following directed graph using the Floyd–Warshall algorithm. Your answer should include the matrices D^j and E^j for every j from 0 to 4.



3. For the directed graph in the previous problem (problem 2), write the steps necessary to reconstruct the shortest path from node 4 to node 1 using the matrix E^4.

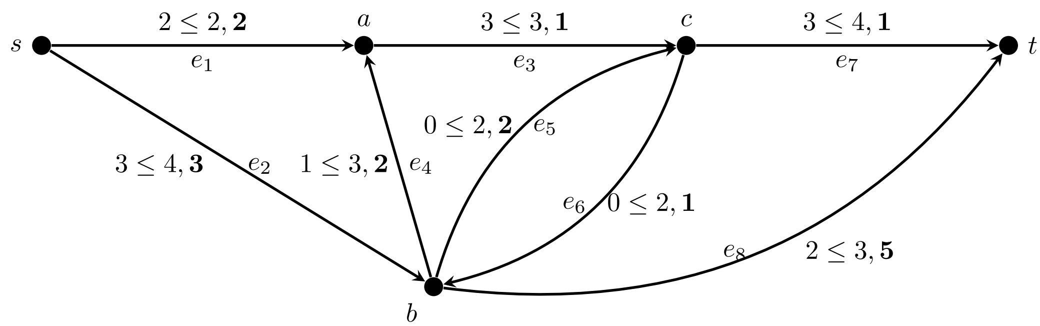
4. Let N be the network consisting of the directed graph with edges $E = [sa, sb, ac, ba, bc, cb, ct, bt]$, capacities $b = [2, 4, 3, 3, 2, 2, 4, 3]^T$, costs $c^T = [2, 3, 1, 2, 2, 1, 1, 5]$ and let $f = [2, 3, 3, 1, 0, 0, 3, 2]^T$ be a flow. This network and flow is drawn below. Our aim is to construct a min-cost flow with value 5.

   (a) Compute the cost of the given flow.

   (b) Draw the incremental weighted flow network $N'(f)$ and find a negative cost f^r cycle in $N'(f)$ (you do not need to use the Floyd-Warshall algorithm for this step).

   (c) Then find the maximum θ such that $f^* = f + \theta f^r$ is a feasible flow.

   (d) Determine if the flow $f^* = f + \theta f^r$ is a minimum cost flow with value 5.



5. Let $G = (V, E)$ be an undirected graph. A set $U \subseteq V$ is an independent set in G, if there does not exist an edge $\{v_i, v_j\} \in E$ such that both $v_i \in U$ and $v_j \in U$. Formulate the problem of finding a maximum size independent set in G as an integer linear program.